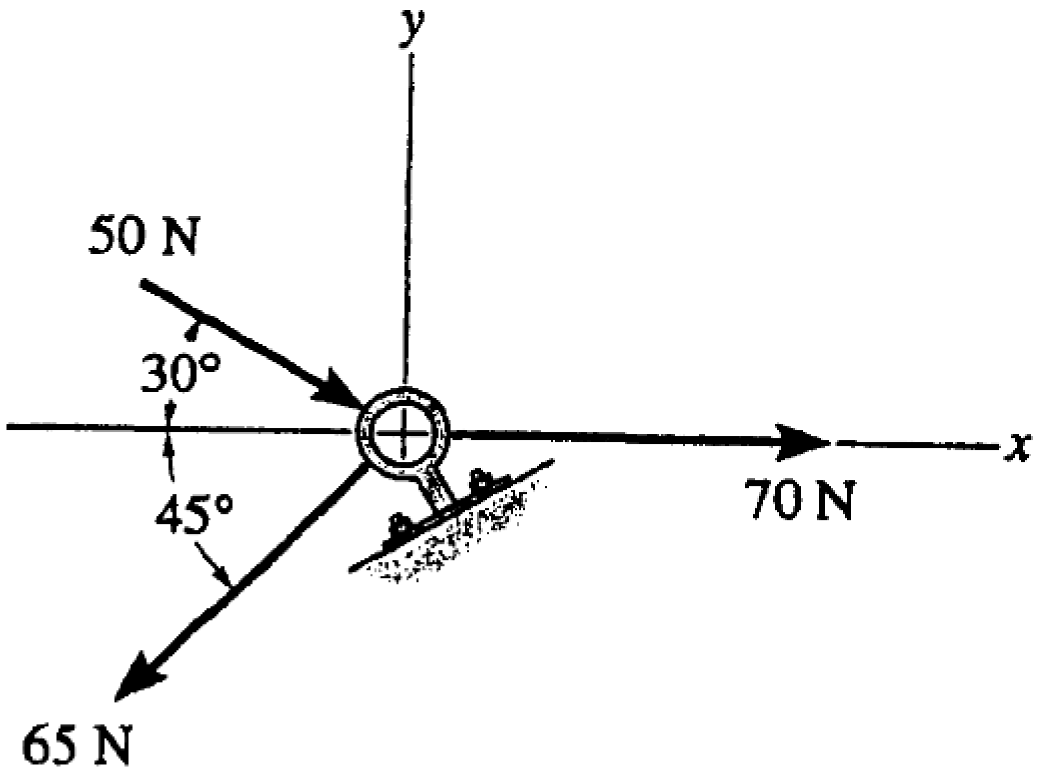
HW#2

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.

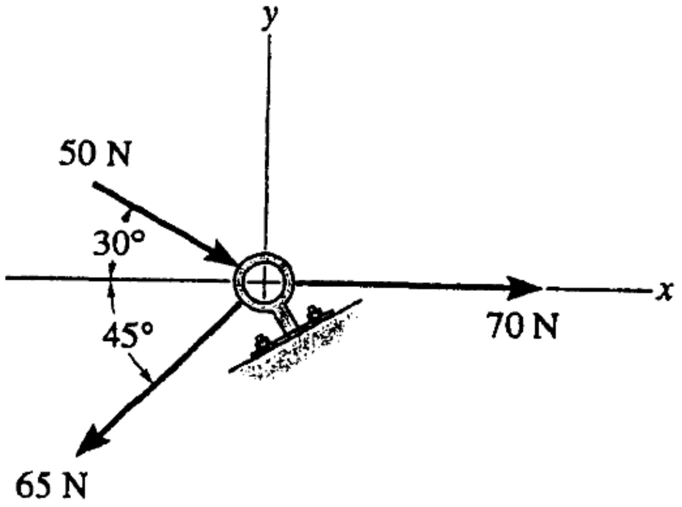


$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 70 + 50\cos 30^\circ - 65\cos 45^\circ = 67.34 \text{ N}$$

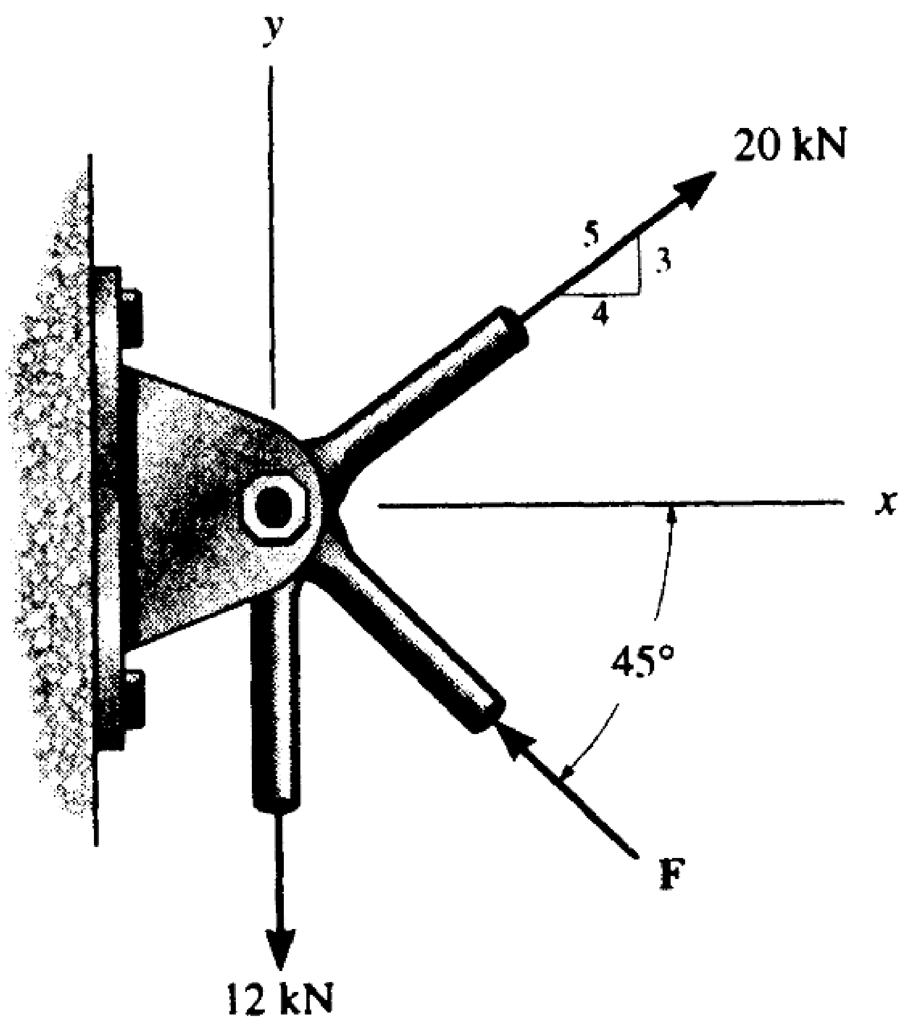
$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -50\sin 30^\circ - 65\sin 45^\circ = -70.96 \text{ N}$$

$$F_R = \sqrt{(67.34)^2 + (-70.96)^2} = 97.8 \text{ N}$$

$$\theta = \tan^{-1}\frac{70.96}{67.34} = 46.5^\circ$$



Determine the magnitude of force F so that the resultant FR of the three forces is as small as possible.

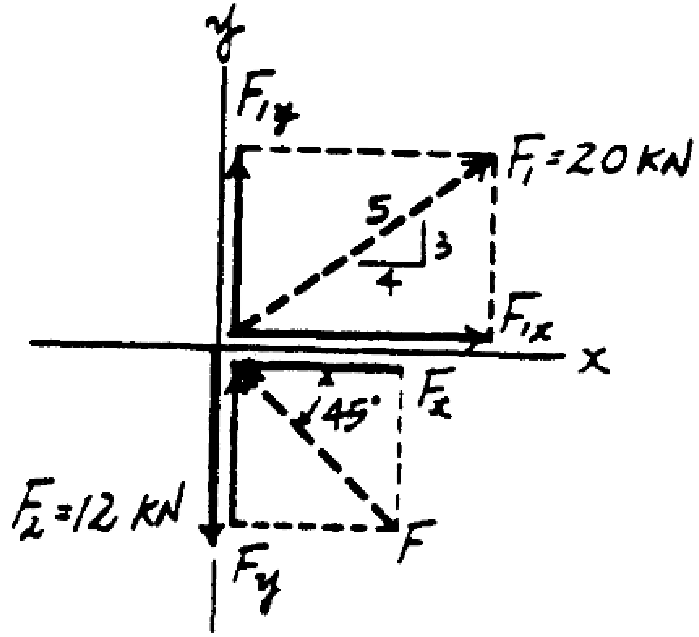


$\xrightarrow{+} F_{R_x} = \Sigma F_x;$ $\quad F_{R_x} = 20\left(\frac{4}{5}\right) - F\cos 45°$

$$= 16.0 - 0.7071F \rightarrow$$

$+\uparrow F_{R_y} = \Sigma F_y;$ $\quad F_{R_y} = 20\left(\frac{3}{5}\right) - 12 + F\sin 45°$

$$= 0.7071F \uparrow$$

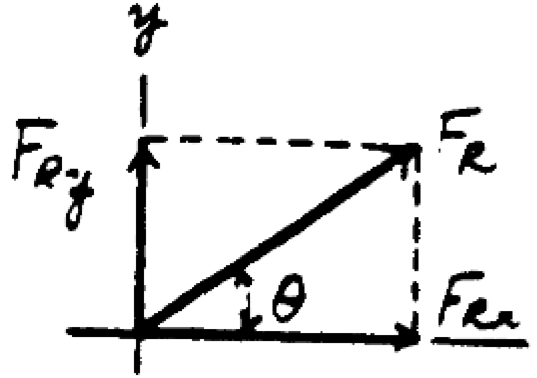


$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$

$$= \sqrt{(16.0 - 0.7071F)^2 + (0.7071F)^2}$$

$$= \sqrt{F^2 - 22.63F + 256}$$

[1]



$$F_R^2 = F^2 - 22.63F + 256$$

$$2F_R \frac{dF_R}{dF} = 2F - 22.63$$

[2]

$$\left(F_R \frac{d^2 F_R}{dF^2} + \frac{dF_R}{dF} \times \frac{dF_R}{dF}\right) = 1 \qquad [3]$$

In order to obtain the *minimum* resultant force $\mathbf{F}_R$, $\frac{dF_R}{dF} = 0$. From Eq.[2]

$$2F_R \frac{dF_R}{dF} = 2F - 22.63 = 0$$

$$F = 11.31 \text{ kN} = 11.3 \text{ kN}$$

Substitute $F = 11.31$ kN into Eq.[1], we have

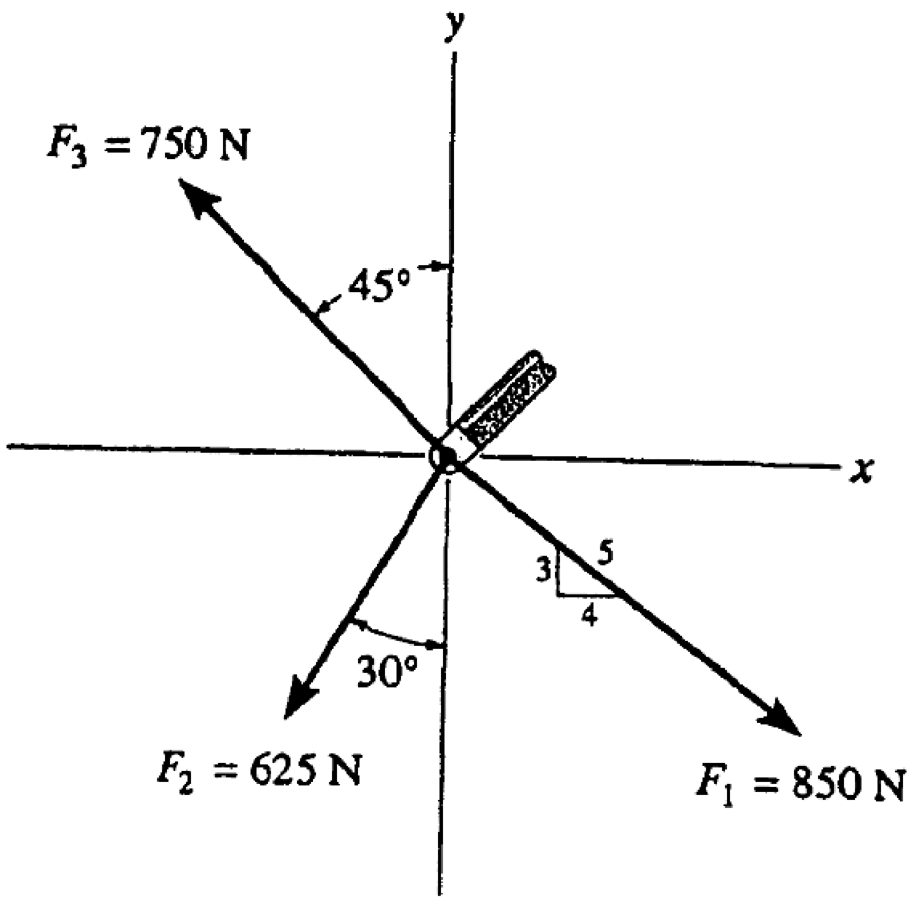
$$F_R = \sqrt{11.31^2 - 22.63(11.31) + 256} = \sqrt{128} \text{ kN}$$

Substitute $F_R = \sqrt{128}$ kN with $\frac{dF_R}{dF} = 0$ into Eq.[3], we have $\left(\sqrt{128}\frac{d^2 F_R}{dF^2} + 0\right) = 1$

$$\frac{d^2 F_R}{dF^2} = 0.0884 > 0$$

Hence, $F = 11.3$ kN is indeed producing a minimum resultant force.

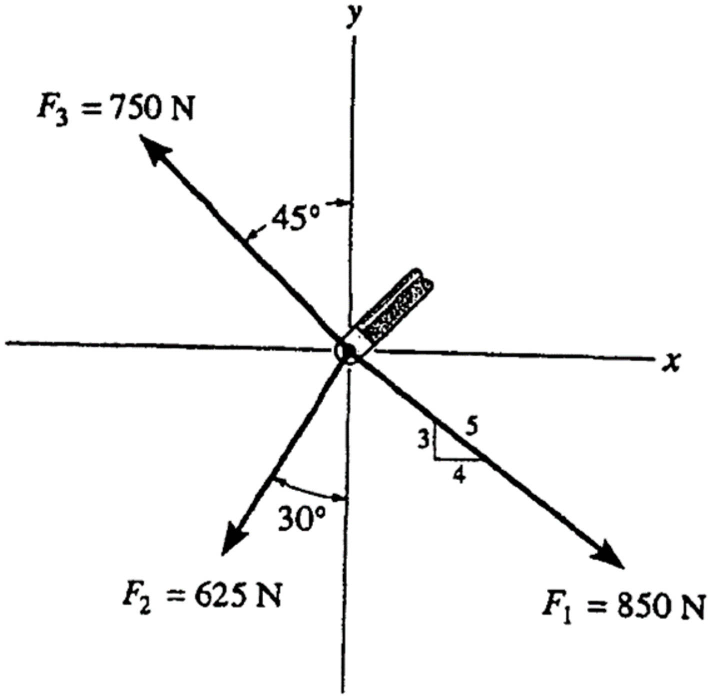
Determine the magnitude of the resultant force and its direction, measured counter clockwise from the positive x axis.



$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = \frac{4}{5}(850) - 625 \sin 30° - 750 \sin 45° = -162.8 \text{ N}$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -\frac{3}{5}(850) - 625 \cos 30° + 750 \cos 45° = -520.9 \text{ N}$$

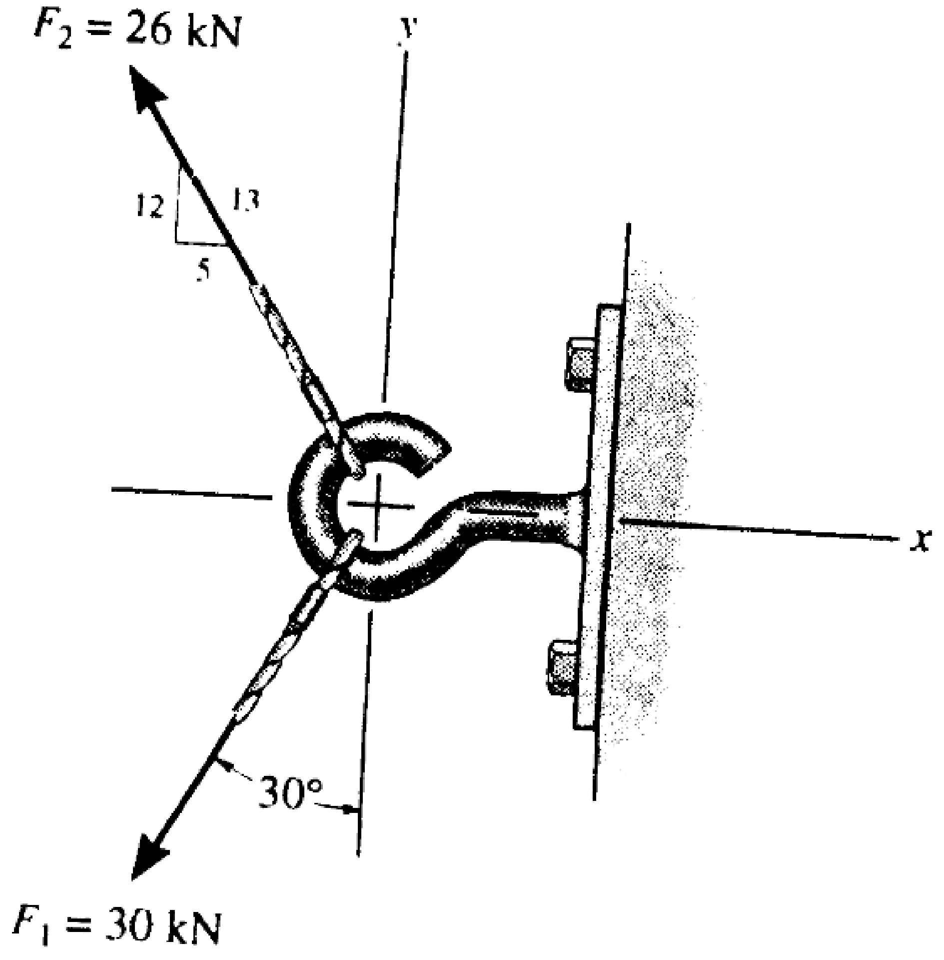
$$F_R = \sqrt{(-162.8)^2 + (-520.9)^2} = 546 \text{ N}$$



$$\phi = \tan^{-1}\left[\frac{-520.9}{-162.8}\right] = 72.64°$$

$$\theta = 180° + 72.64° = 253°$$

Express F1 and F2 as Cartesian vectors.

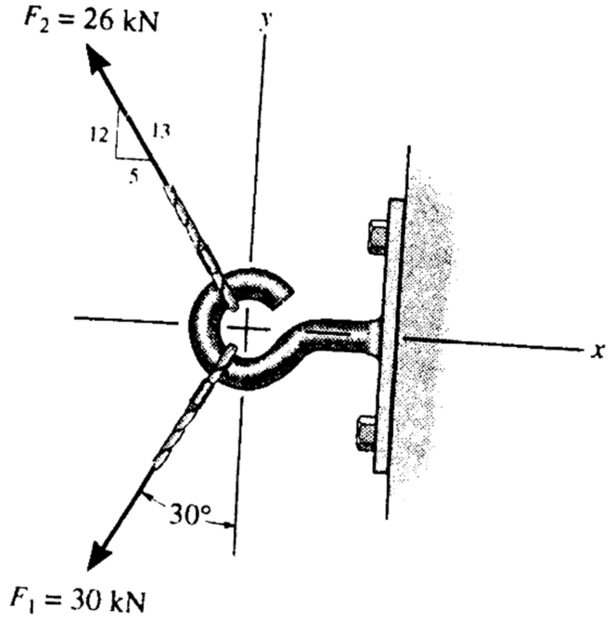


$$\mathbf{F}_1 = -30 \sin 30° \, \mathbf{i} - 30 \cos 30° \, \mathbf{j}$$

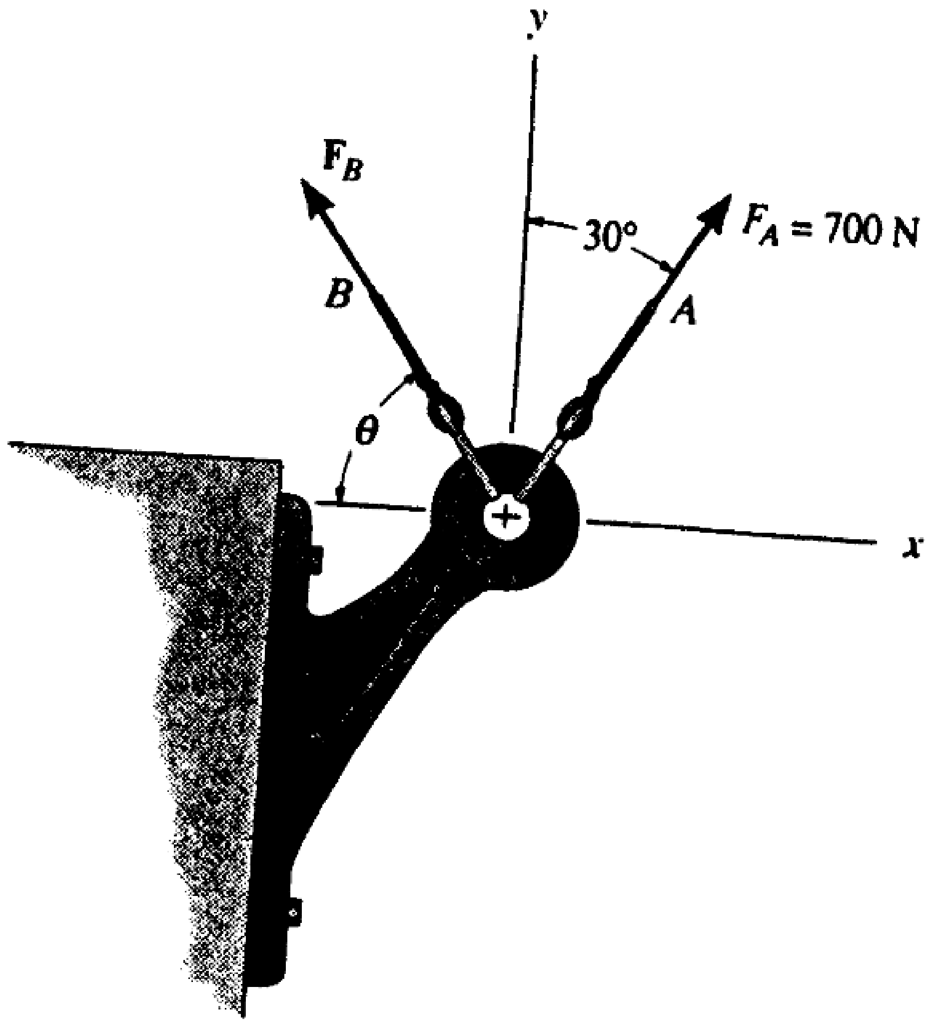
$$= \{-15.0 \, \mathbf{i} - 26.0 \, \mathbf{j}\} \text{ kN}$$

$$\mathbf{F}_2 = -\frac{5}{13}(26) \, \mathbf{i} + \frac{12}{13}(26) \, \mathbf{j}$$

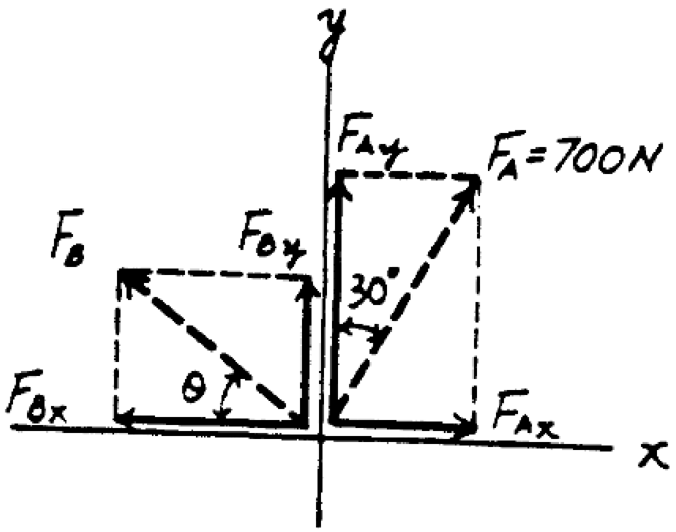
$$= \{-10.0 \, \mathbf{i} + 24.0 \, \mathbf{j}\} \text{ kN}$$



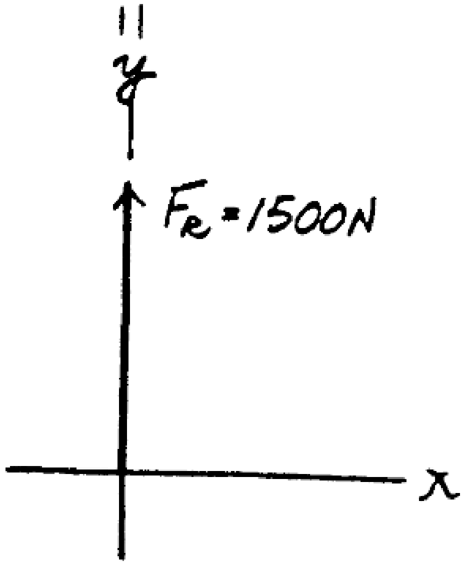
Determine the magnitude and orientation θ of FB so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



$\xrightarrow{+} F_{R_x} = \Sigma F_x;$ $$0 = 700\sin 30° - F_B \cos\theta$$
$$F_B \cos\theta = 350 \qquad [1]$$



$+\uparrow F_{R_y} = \Sigma F_y;$ $$1500 = 700\cos 30° + F_B \sin\theta$$
$$F_B \sin\theta = 893.8 \qquad [2]$$



Solving Eq. [1] and [2] yields

$$\theta = 68.6° \qquad F_B = 960 \text{ N}$$

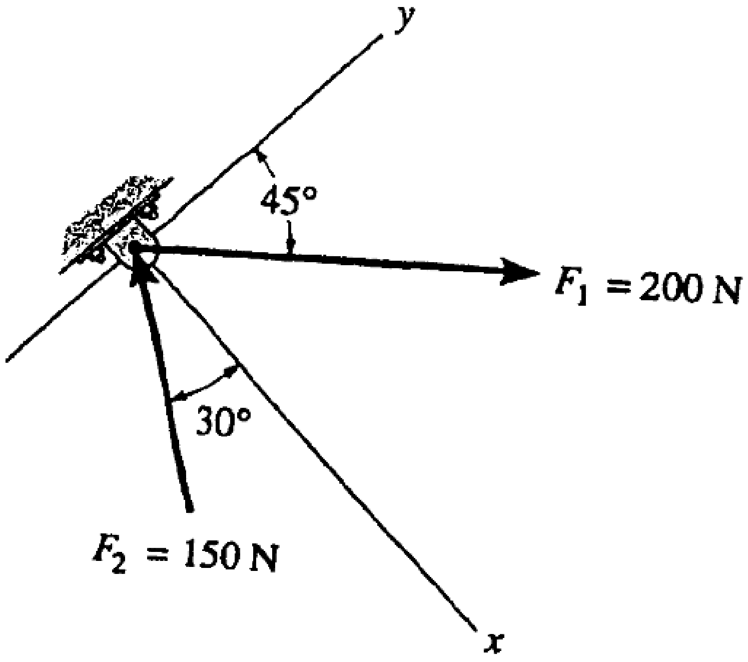
Determine the x and y components of Fl and F2.

$$F_{1x} = 200 \sin 45° = 141 \text{ N}$$

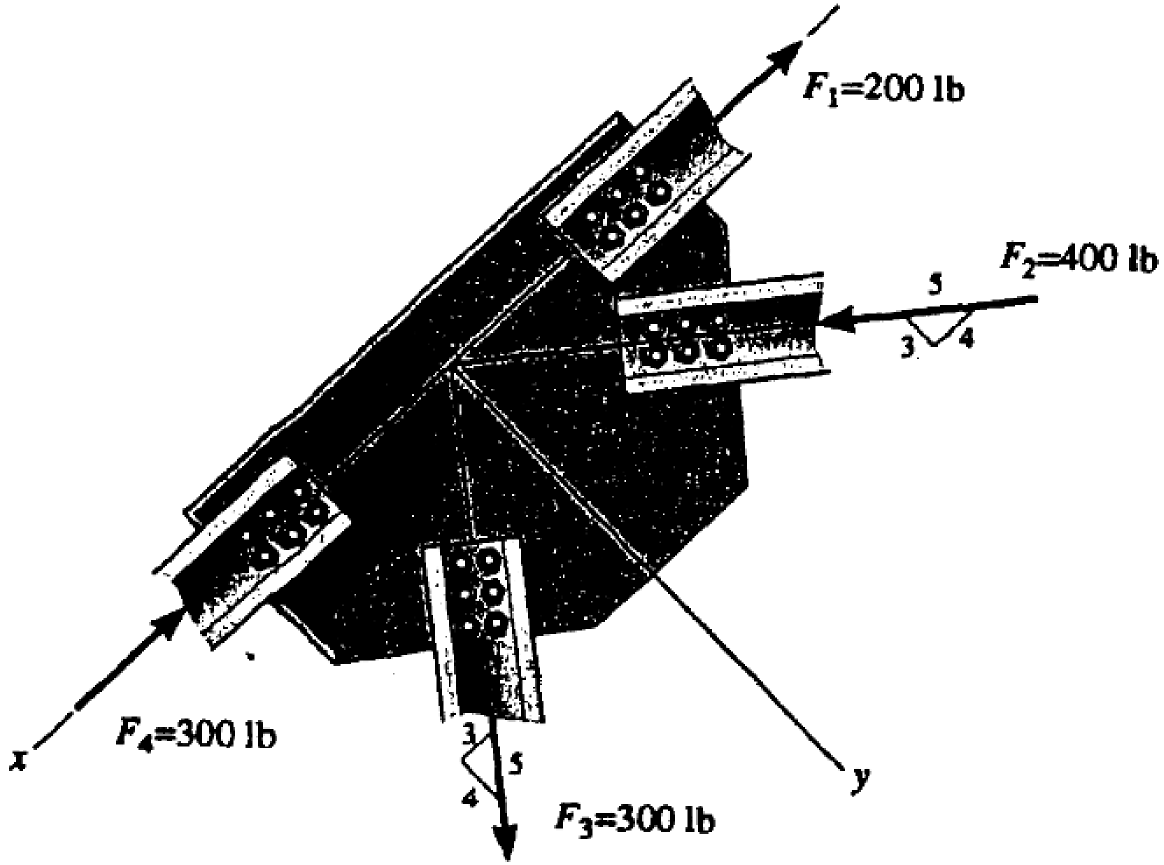
$$F_{1y} = 200 \cos 45° = 141 \text{ N}$$

$$F_{2x} = -150 \cos 30° = -130 \text{ N}$$

$$F_{2y} = 150 \sin 30° = 75 \text{ N}$$



Determine the x and y components of each force acting on the gusset plate of the bridge truss. Show that the resultant force is zero.



$$F_{1x} = -200 \text{ lb} \qquad F_{1y} = 0$$

$$F_{2x} = 400\left(\frac{4}{5}\right) = 320 \text{ lb} \qquad F_{2y} = -400\left(\frac{3}{5}\right) = -240 \text{ lb}$$

$$F_{3x} = 300\left(\frac{3}{5}\right) = 180 \text{ lb} \qquad F_{3y} = 300\left(\frac{4}{5}\right) = 240 \text{ lb}$$

$$F_{4x} = -300 \text{ lb} \qquad F_{4y} = 0$$

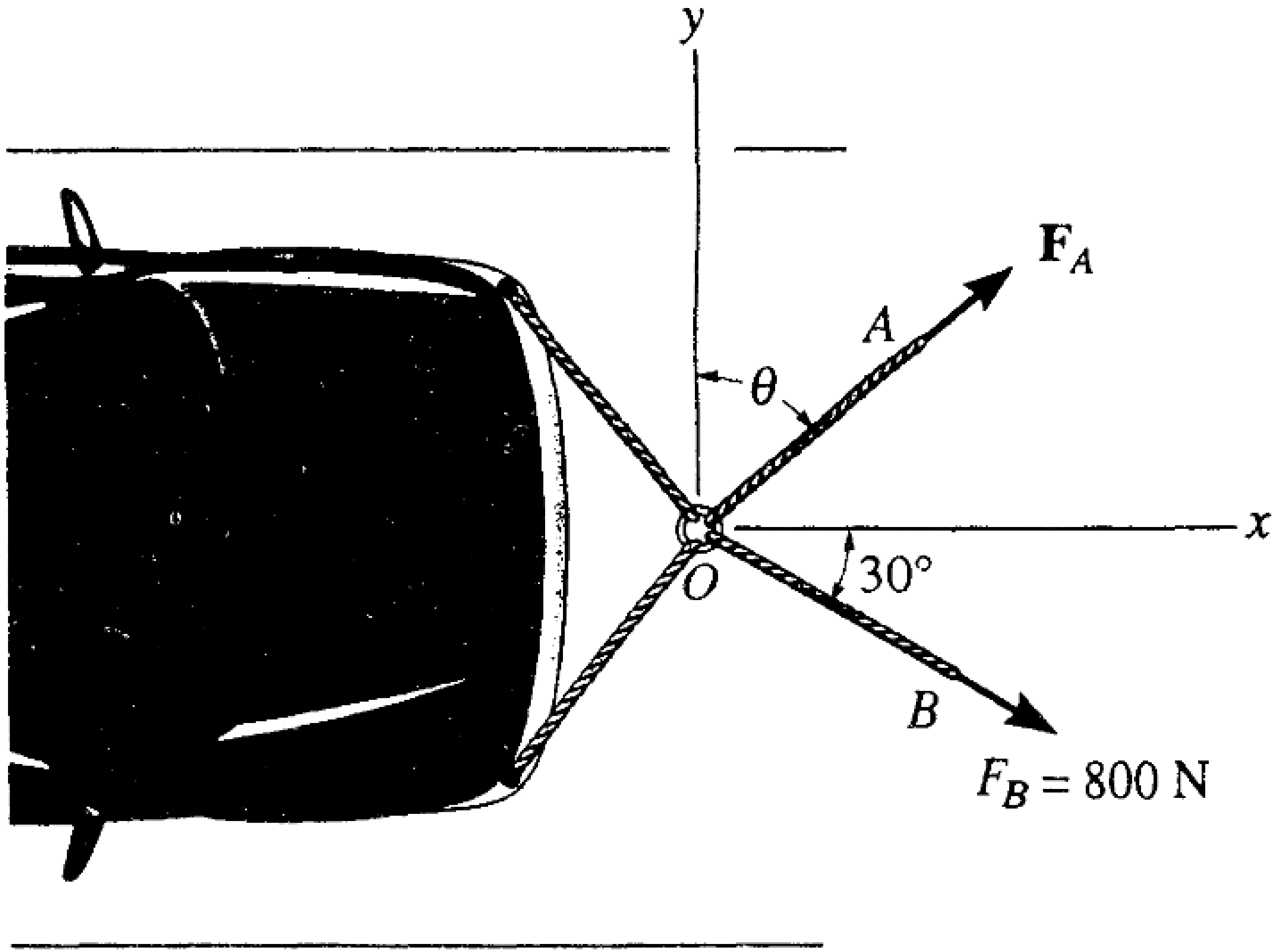
$$F_{Rx} = F_{1x} + F_{2x} + F_{3x} + F_{4x}$$

$$F_{Rx} = -200 + 320 + 180 - 300 = 0$$

$$F_{Ry} = F_{1y} + F_{2y} + F_{3y} + F_{4y}$$

$$F_{Ry} = 0 - 240 + 240 + 0 = 0$$

Determine the magnitude and direction θ of FA so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



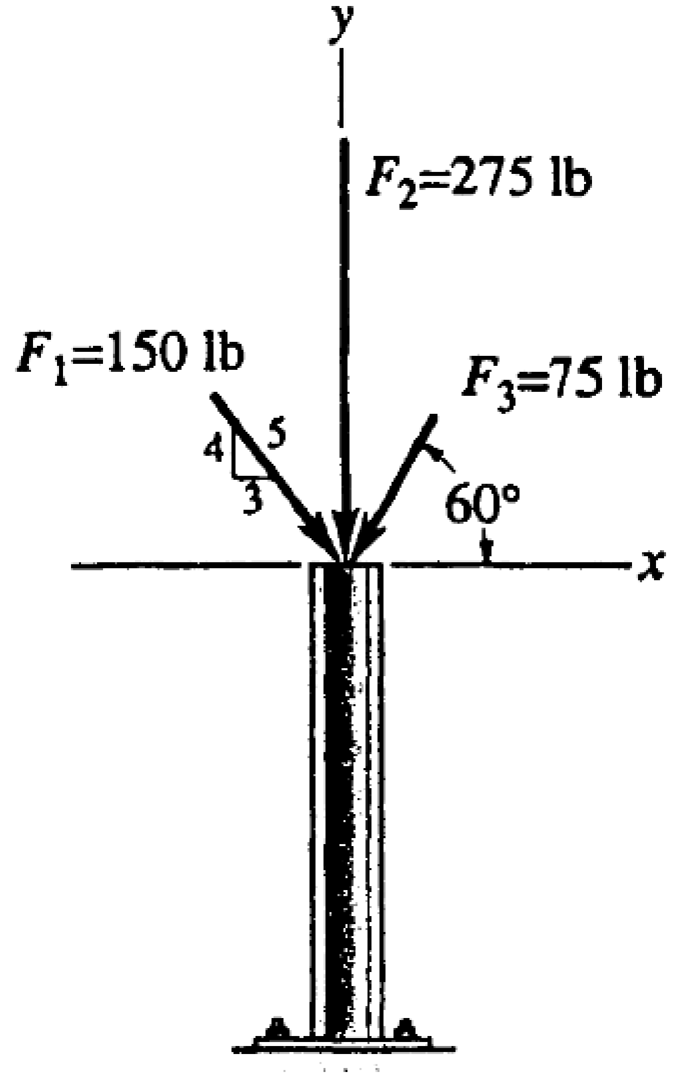
$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = F_A \sin\theta + 800 \cos 30° = 1250$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = F_A \cos\theta - 800 \sin 30° = 0$$

$$\theta = 54.3°$$

$$F_A = 686 \text{ N}$$

Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.



$$\mathbf{F}_1 = 150\left(\frac{3}{5}\right)\mathbf{i} - 150\left(\frac{4}{5}\right)\mathbf{j}$$

$$\mathbf{F}_1 = \{90\mathbf{i} - 120\mathbf{j}\}\ \text{lb}$$

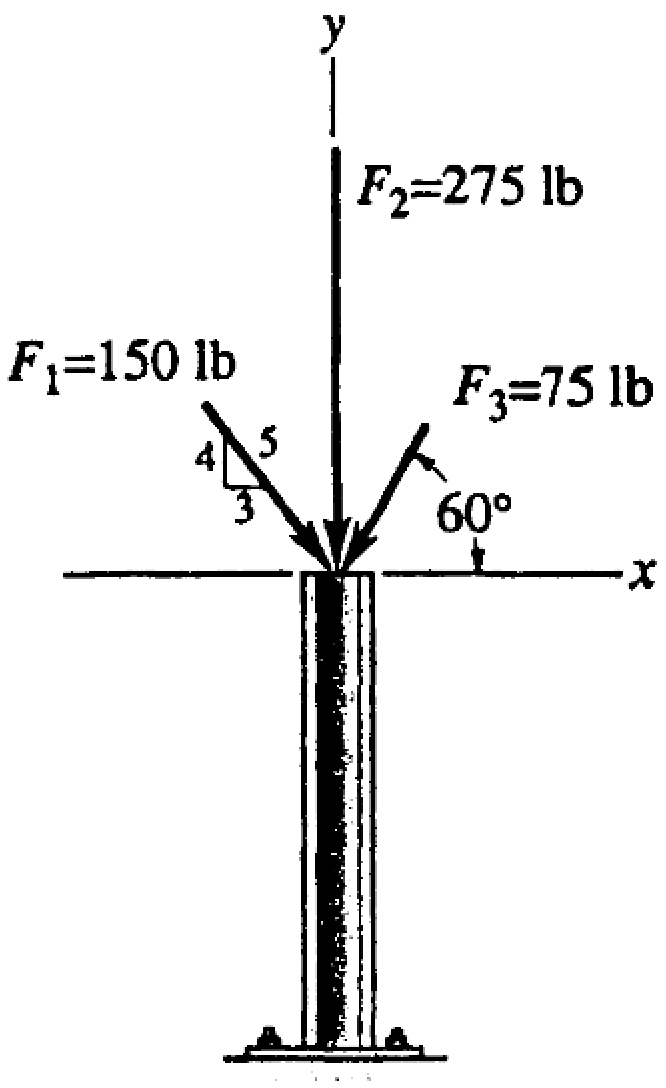
$$\mathbf{F}_2 = \{-275\mathbf{j}\}\ \text{lb}$$

$$\mathbf{F}_3 = -75\cos 60°\mathbf{i} - 75\sin 60°\mathbf{j}$$

$$\mathbf{F}_3 = \{-37.5\mathbf{i} - 65.0\mathbf{j}\}\ \text{lb}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{52.5\mathbf{i} - 460\mathbf{j}\}\ \text{lb}$$

$$F_R = \sqrt{(52.5)^2 + (-460)^2} = 463\ \text{lb}$$



Determine the magnitude and coordinate direction angles of F1 = (6Oi - 50j + 40k) N and F2 = (-40i - 85j+ 30k) N. Sketch each force on an x, y, z reference.

$$\mathbf{F}_1 = 60\,\mathbf{i} - 50\,\mathbf{j} + 40\,\mathbf{k}$$

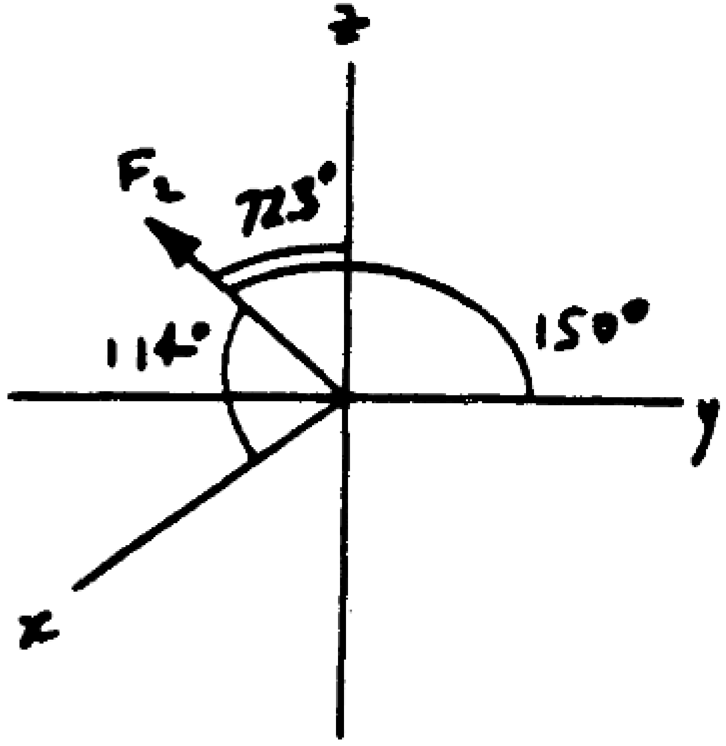
$$F_1 = \sqrt{(60)^2 + (-50)^2 + (40)^2} = 87.750 = 87.7\text{ N}$$

$$\alpha_1 = \cos^{-1}\left(\frac{60}{87.750}\right) = 46.9^\circ \qquad \beta_1 = \cos^{-1}\left(\frac{-50}{87.750}\right) = 125^\circ$$

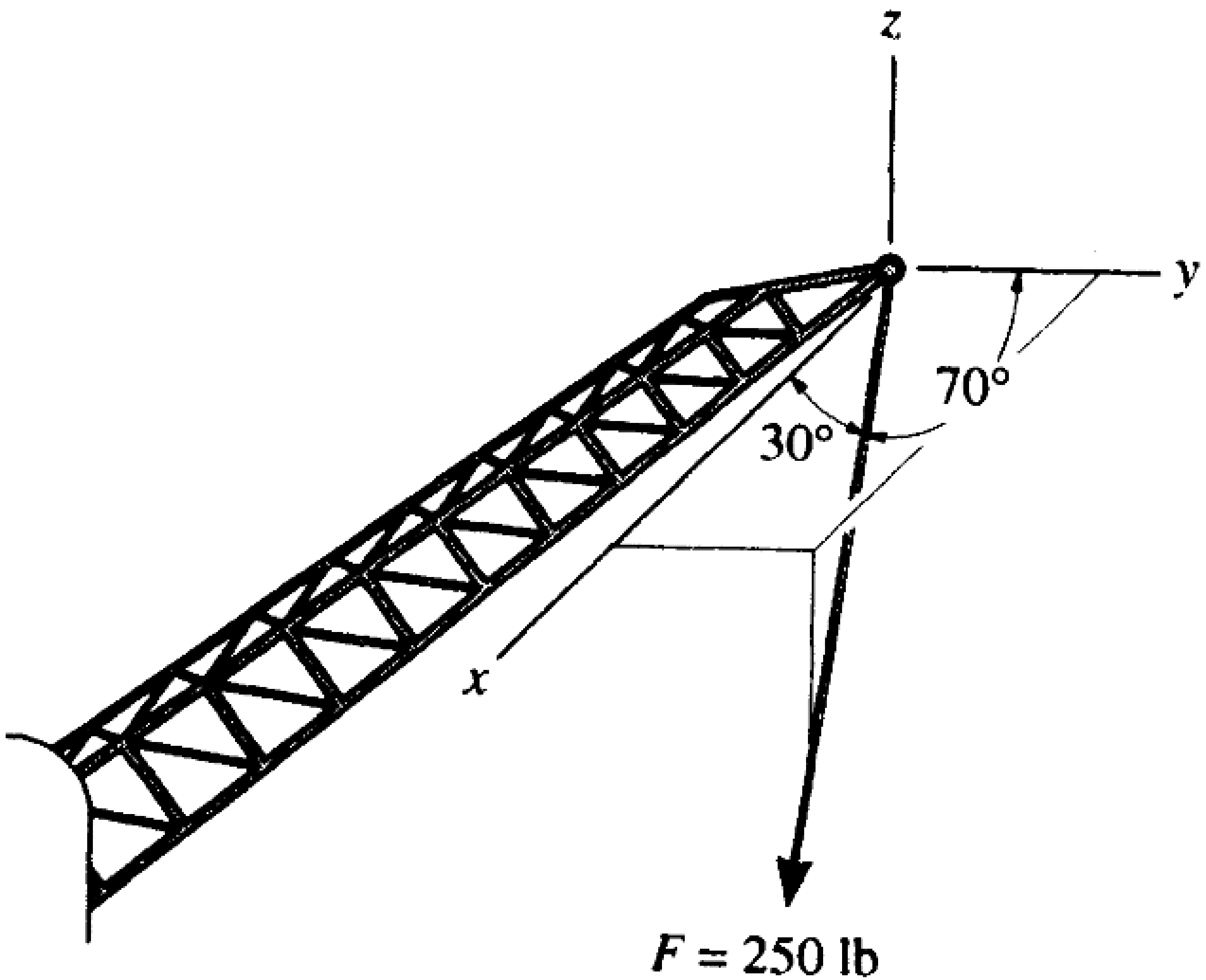
$$\gamma_1 = \cos^{-1}\left(\frac{40}{87.750}\right) = 62.9^\circ$$

$$\mathbf{F}_2 = -40\,\mathbf{i} - 85\,\mathbf{j} + 30\,\mathbf{k} \qquad \alpha_2 = \cos^{-1}\left(\frac{-40}{98.615}\right) = 114^\circ$$

$$\beta_2 = \cos^{-1}\left(\frac{-85}{98.615}\right) = 150^\circ \qquad \gamma_2 = \cos^{-1}\left(\frac{30}{98.615}\right) = 72.3^\circ$$


z
F_2
114°
150°
y
x

The cable at the end of the crane boom exerts a force of 250 Ib on the boom as shown. Express F as a Cartesian vector.



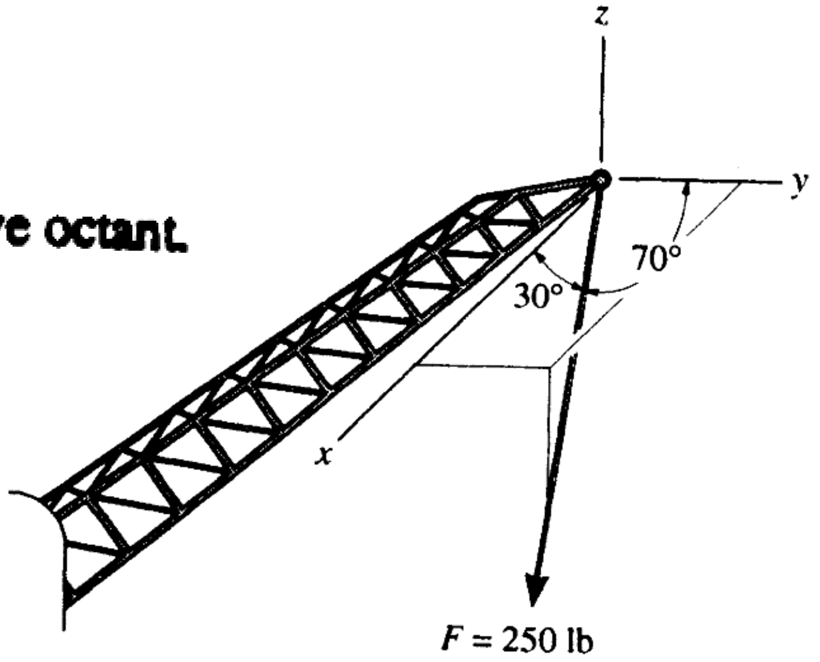
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 30° + \cos^2 70° + \cos^2\gamma = 1$$

$$\cos\gamma = \pm 0.3647$$

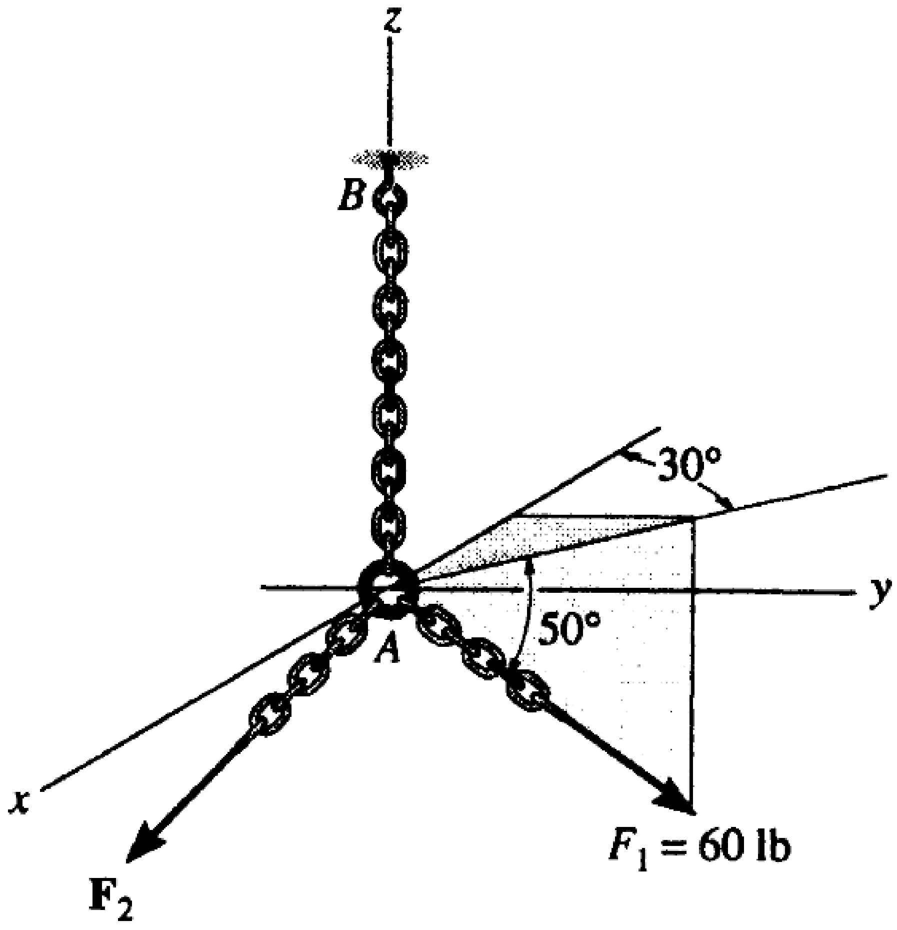
$$\gamma = 68.61° \quad \text{or} \quad 111.39°$$

By inspection, $\gamma = 111.39°$ since the force **F** is directed in negative octant.



$$\mathbf{F} = 250\{\cos 30°\mathbf{i} + \cos 70°\mathbf{j} + \cos 111.39°\}\ \text{lb}$$
$$= \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\}\ \text{lb}$$

The two forces F1 and F2 acting at A have a resultant force of FR = (-l00k) lb. Determine the magnitude and coordinate direction angles of F2•



$$\mathbf{F}_R = \{-100\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_1 = 60\{-\cos 50^\circ \cos 30^\circ \mathbf{i} + \cos 50^\circ \sin 30^\circ \mathbf{j} - \sin 50^\circ \mathbf{k}\}$$
$$= \{-33.40\mathbf{i} + 19.28\mathbf{j} - 45.96\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = \{F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j} + F_{2_z}\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$-100\mathbf{k} = \left\{\left(F_{2_x} - 33.40\right)\mathbf{i} + \left(F_{2_y} + 19.28\right)\mathbf{j} + \left(F_{2_z} - 45.96\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$F_{2_x} - 33.40 = 0 \qquad F_{2_x} = 33.40 \text{ lb}$$
$$F_{2_y} + 19.28 = 0 \qquad F_{2_y} = -19.28 \text{ lb}$$
$$F_{2_z} - 45.96 = -100 \qquad F_{2_z} = -54.04 \text{ lb}$$

The magnitude of force $\mathbf{F}_2$ is

$$F_2 = \sqrt{F_{2_x}^2 + F_{2_y}^2 + F_{2_z}^2}$$

$$= \sqrt{33.40^2 + (-19.28)^2 + (-54.04)^2}$$
$$= 66.39 \text{ lb} = 66.4 \text{ lb}$$

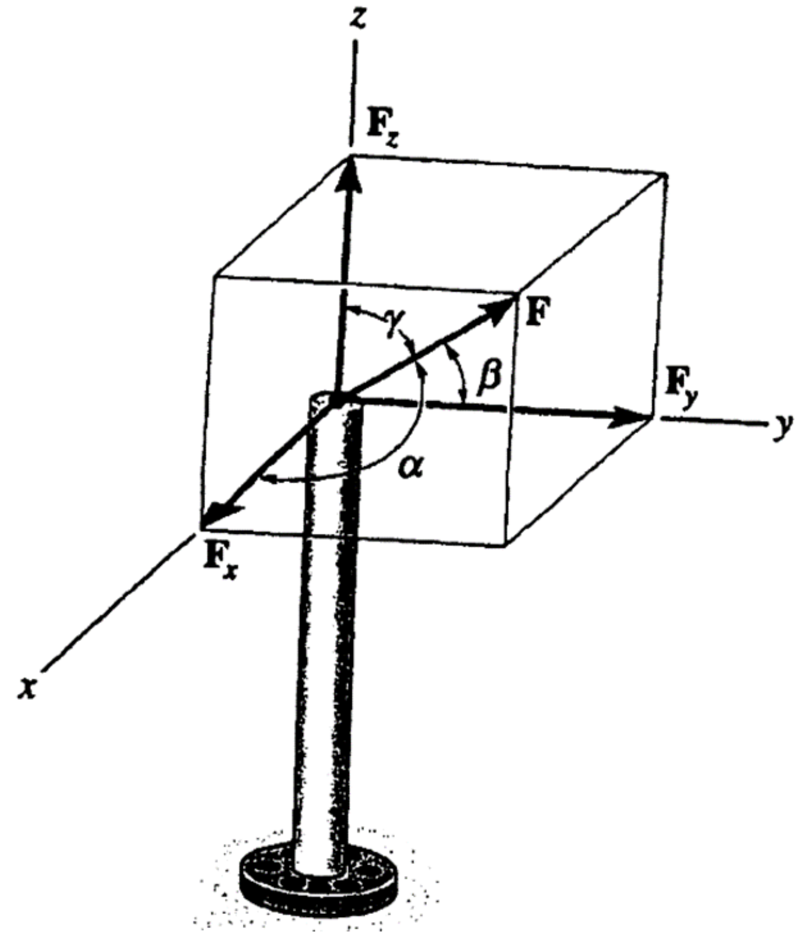
The coordinate direction angles for $\mathbf{F}_2$ are

$$\cos\alpha = \frac{F_{2_x}}{F_2} = \frac{33.40}{66.39} \qquad \alpha = 59.8°$$

$$\cos\beta = \frac{F_{2_y}}{F_2} = \frac{-19.28}{66.39} \qquad \beta = 107°$$

$$\cos\gamma = \frac{F_{2_z}}{F_2} = \frac{-54.04}{66.39} \qquad \gamma = 144°$$

The pole is subjected to the force F, which has components acting along the x, y, Z axes as shown. If the magnitude of F is 3 kN, and β = 30°· and γ = 75°, determine the magnitudes of its three components.



$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

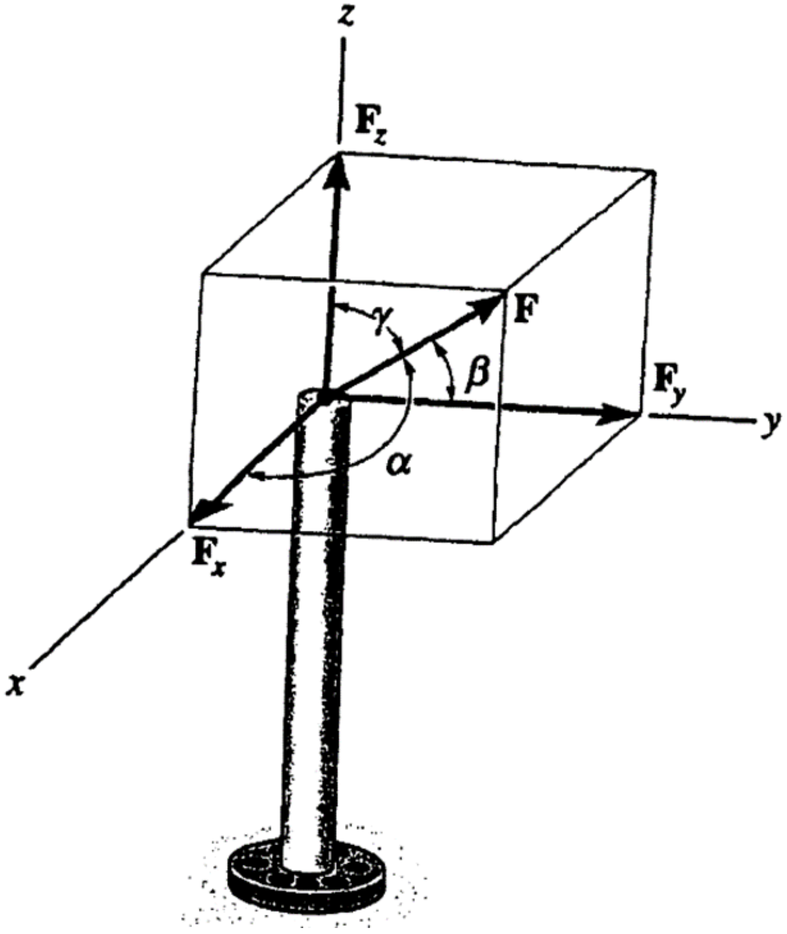
$$\cos^2\alpha + \cos^2 30° + \cos^2 75° = 1$$

$$\alpha = 64.67°$$

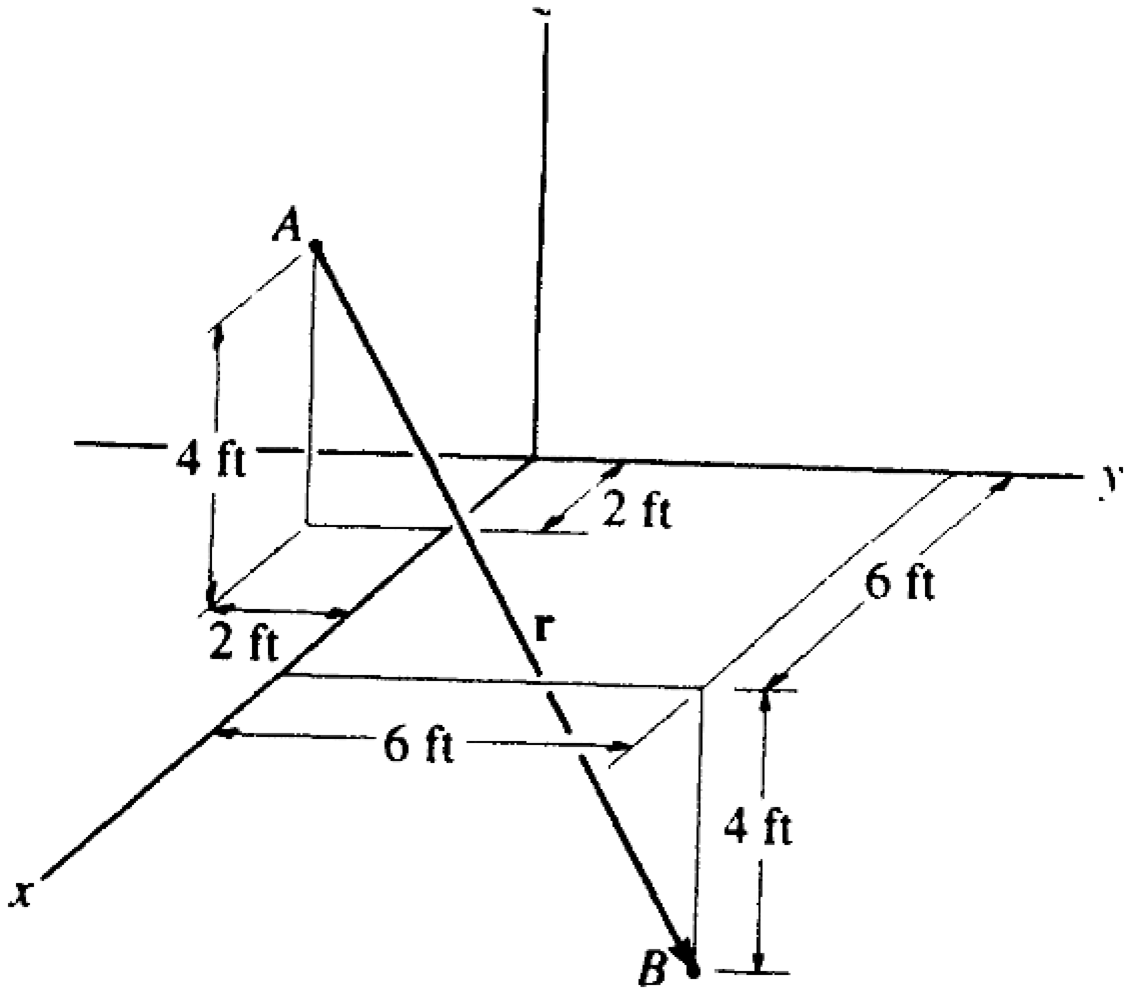
$$F_x = 3\cos 64.67° = 1.28 \text{ kN}$$

$$F_y = 3\cos 30° = 2.60 \text{ kN}$$

$$F_z = 3\cos 75° = 0.776 \text{ kN}$$



Express the position vector r in Cartesian vector form; then determine its magnitude and coordinate direction angles.



Position Vector :

$$\mathbf{r} = \{(6-2)\mathbf{i} + [6-(-2)]\mathbf{j} + (-4-4)\mathbf{k}\}\ \text{ft}$$
$$= \{4\mathbf{i} + 8\mathbf{j} - 8\mathbf{k}\}\ \text{ft}$$

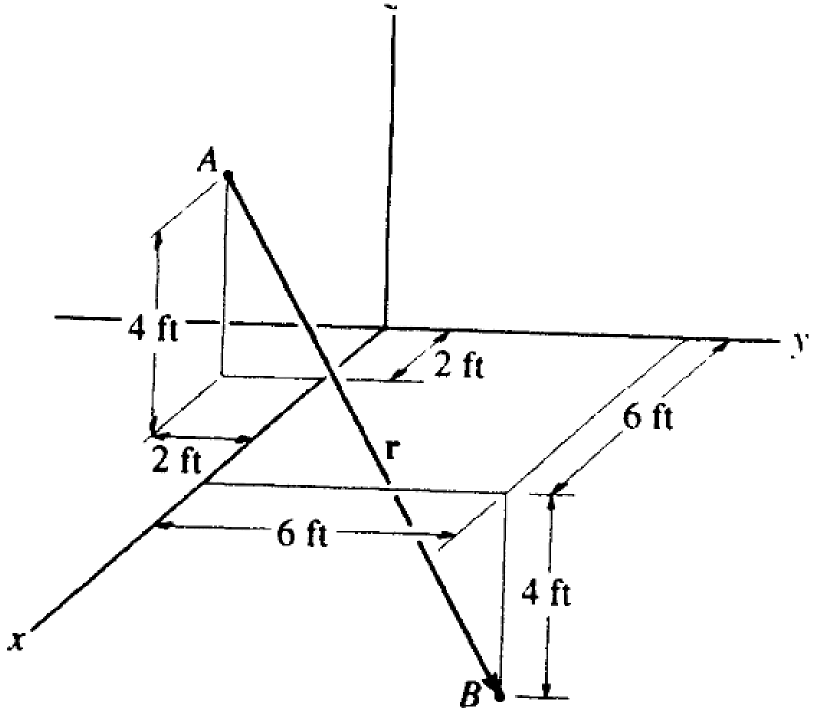
The magnitude of **r** is

$$r = \sqrt{4^2 + 8^2 + (-8)^2} = 12.0\ \text{ft}$$

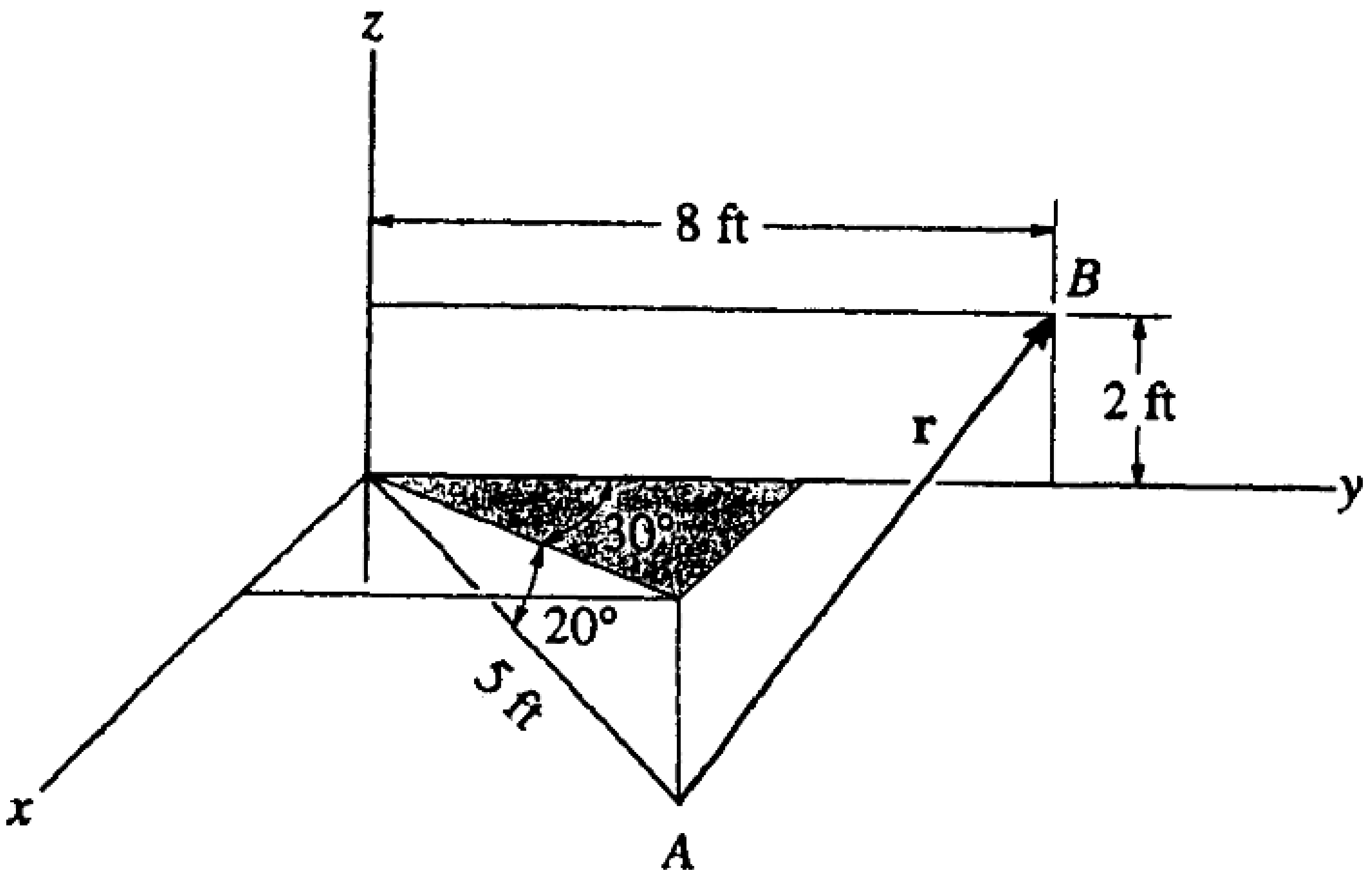
$$\cos\alpha = \frac{4}{12.0} \qquad \alpha = 70.5^\circ$$

$$\cos\beta = \frac{8}{12.0} \qquad \beta = 48.2^\circ$$

$$\cos\gamma = \frac{-8}{12.0} \qquad \gamma = 132^\circ$$



Express the position vector r in Cartesian vector form; then determine its magnitude and coordinate direction angles.



$$\mathbf{r} = (-5\cos 20^\circ \sin 30^\circ \mathbf{i} + (8 - 5\cos 20^\circ \cos 30^\circ)\mathbf{j} + (2 + 5\sin 20^\circ)\mathbf{k})$$

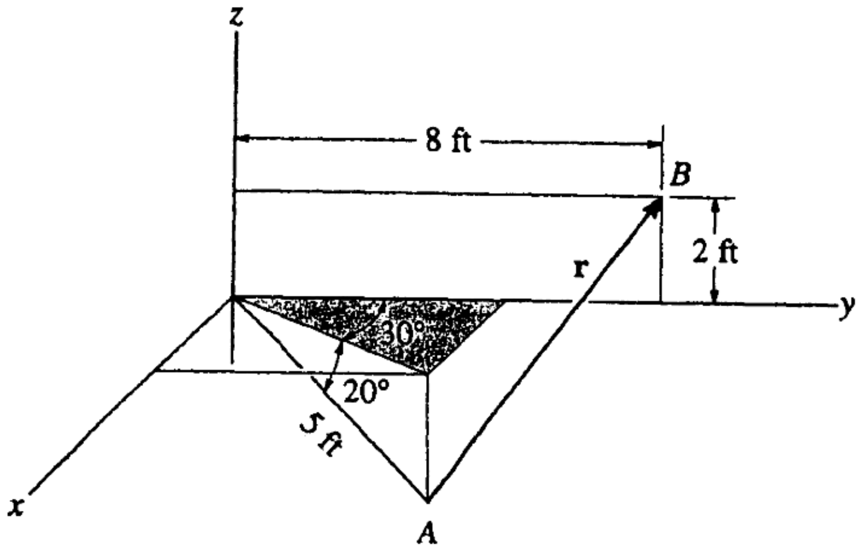
$$\mathbf{r} = \{-2.35\mathbf{i} + 3.93\mathbf{j} + 3.71\mathbf{k}\} \text{ ft}$$

$$r = \sqrt{(-2.35)^2 + (3.93)^2 + (3.71)^2} = 5.89 \text{ ft}$$

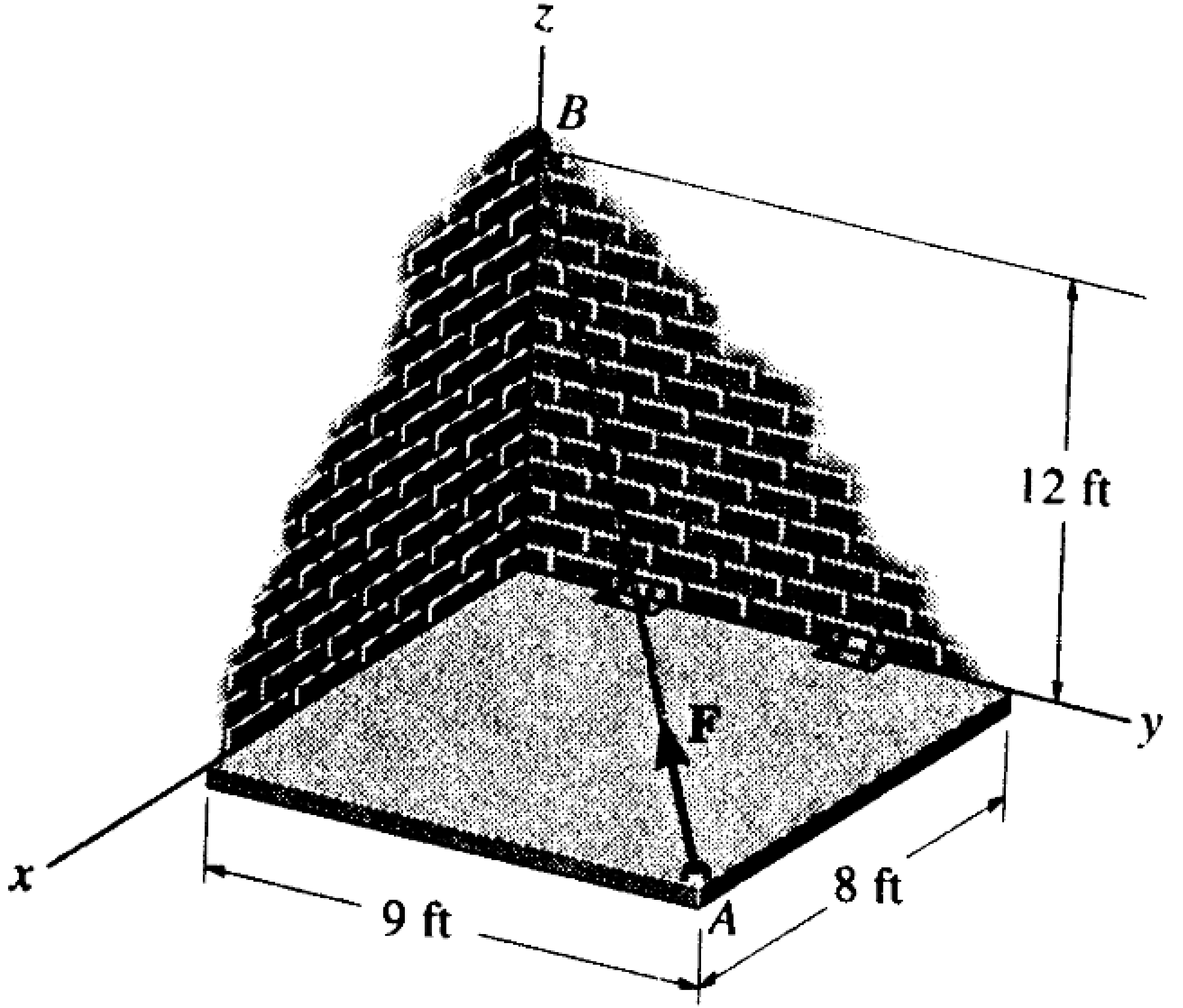
$$\alpha = \cos^{-1}\left(\frac{-2.35}{5.89}\right) = 113^\circ$$

$$\beta = \cos^{-1}\left(\frac{3.93}{5.89}\right) = 48.2^\circ$$

$$\gamma = \cos^{-1}\left(\frac{3.71}{5.89}\right) = 51.0^\circ$$



The hinged plate is supported by the cord AB. If the force in the cord is F = 340 Ib, express this force, directed from A toward B. as a Cartesian vector. What is the length of the cord?



Unit Vector :

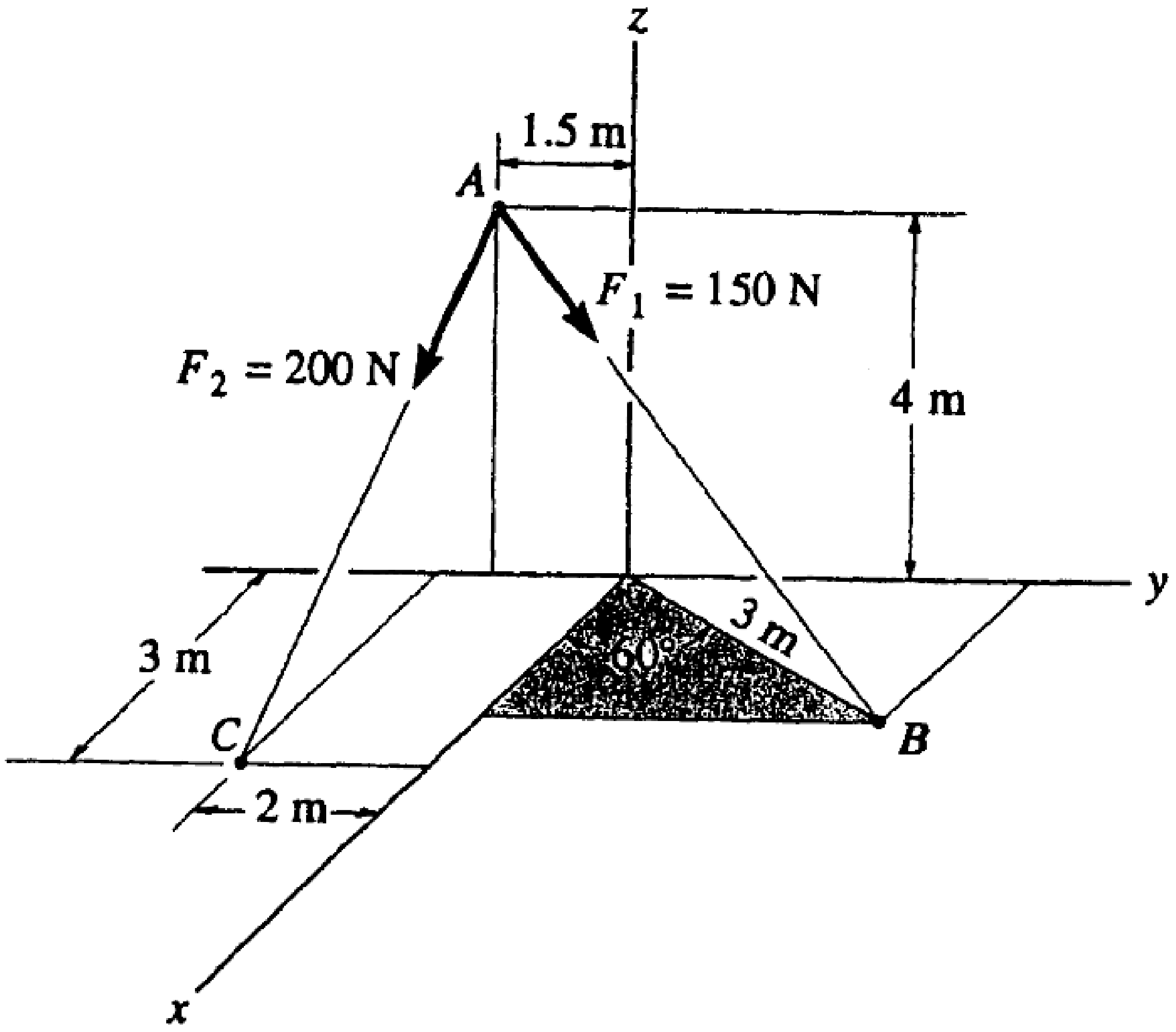
$$\mathbf{r}_{AB} = \{(0-8)\,\mathbf{i} + (0-9)\,\mathbf{j} + (12-0)\,\mathbf{k}\}\ \text{ft}$$
$$= \{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}\}\ \text{ft}$$

$$r_{AB} = \sqrt{(-8)^2 + (-9)^2 + 12^2} = 17.0\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}}{17} = -\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 340\left\{-\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}\right\}\ \text{lb}$$
$$= \{-160\mathbf{i} - 180\mathbf{j} + 240\mathbf{k}\}\ \text{lb}$$

Determine the magnitude and coordinate direction angles of the resultant force acting at point A.



$$\mathbf{r}_{AC} = \{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}\}\ \text{m}$$

$$|\mathbf{r}_{AC}| = \sqrt{3^2 + (-0.5)^2 + (-4)^2} = \sqrt{25.25} = 5.02494$$

$$\mathbf{F}_2 = 200\left(\frac{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}}{5.02494}\right) = (119.4044\mathbf{i} - 19.9007\mathbf{j} - 159.2059\mathbf{k})$$

$$\mathbf{r}_{AB} = (3\cos 60°\mathbf{i} + (1.5 + 3\sin 60°)\mathbf{j} - 4\mathbf{k})$$

$$\mathbf{r}_{AB} = (1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k})$$

$$|\mathbf{r}_{AB}| = \sqrt{(1.5)^2 + (4.0981)^2 + (-4)^2} = 5.9198$$

$$\mathbf{F}_1 = 150\left(\frac{1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k}}{5.9198}\right) = (38.0080\mathbf{i} + 103.8405\mathbf{j} - 101.3548\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (157.4124\mathbf{i} + 83.9398\mathbf{j} - 260.5607\mathbf{k})$$

$$F_R = \sqrt{(157.4124)^2 + (83.9398)^2 + (-260.5607)^2} = 315.7791 = 316 \text{ N}$$

$$\alpha = \cos^{-1}\left(\frac{157.4124}{315.7791}\right) = 60.099° = 60.1°$$

$$\beta = \cos^{-1}\left(\frac{83.9398}{315.7791}\right) = 74.584° = 74.6°$$

$$\gamma = \cos^{-1}\left(\frac{-260.5607}{315.7791}\right) = 145.60° = 146°$$