PART

3

HEAT TRANSFER

CHAPTER

16

MECHANISMS OF HEAT TRANSFER

OBJECTIVES

1- Understand the basic mechanisms of heat transfer, which are conduction, convection, and radiation, and Fourier's law of heat conduction, Newton's law of cooling, and the Stefan– Boltzmann
law of radiation.

2- Identify the mechanisms of heat transfer that occur simultaneously in practice.

3- Develop an awareness of the cost associated with heat losses.

4- Solve various heat transfer problems encountered in practice.

16-1 INTRODUCTION

Heat as the form of energy that can be transferred from **one system to another as a result of temperature difference.**

A thermodynamic analysis is concerned with the *amount of heat transfer as a system undergoes* a process from one equilibrium state to another.

The science that deals with the determination of the *rates of such energy transfers is the heat transfer.*

Heat can be transferred in three different modes: *conduction*, *convection*, and *radiation*.

16-2 CONDUCTION

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.

Conduction can take place in solids, liquids, or gases.

In gases and liquids, conduction is due to the *collisions and diffusion of the* molecules during their random motion.

In solids, it is due to the combination of *vibrations* of the molecules in a lattice and the energy transport by *free electrons*.

***The rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer.* That is:**

$$\text{Rate of heat conduction} \propto \frac{(\text{Area})(\text{Temperature difference})}{\text{Thickness}}$$

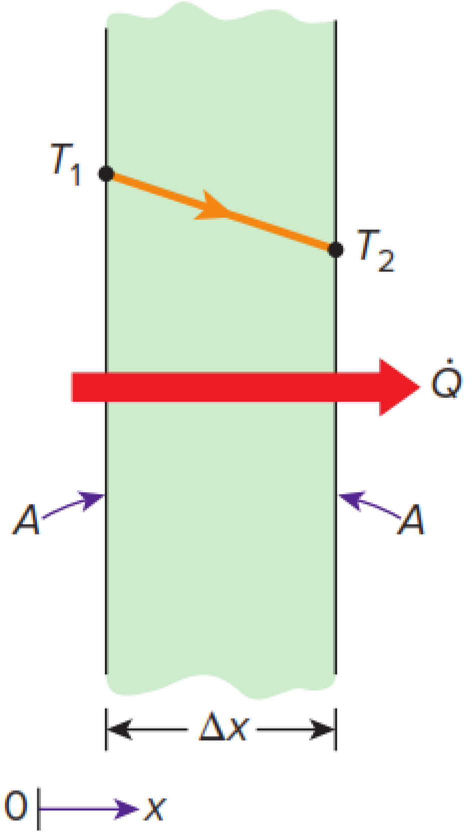


FIGURE 16–1

$$\dot{Q}_{cond} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \qquad (W)$$

where *k is the thermal conductivity of* the material, which is a *measure of the ability of a material to conduct heat* (Fig. 16–2).

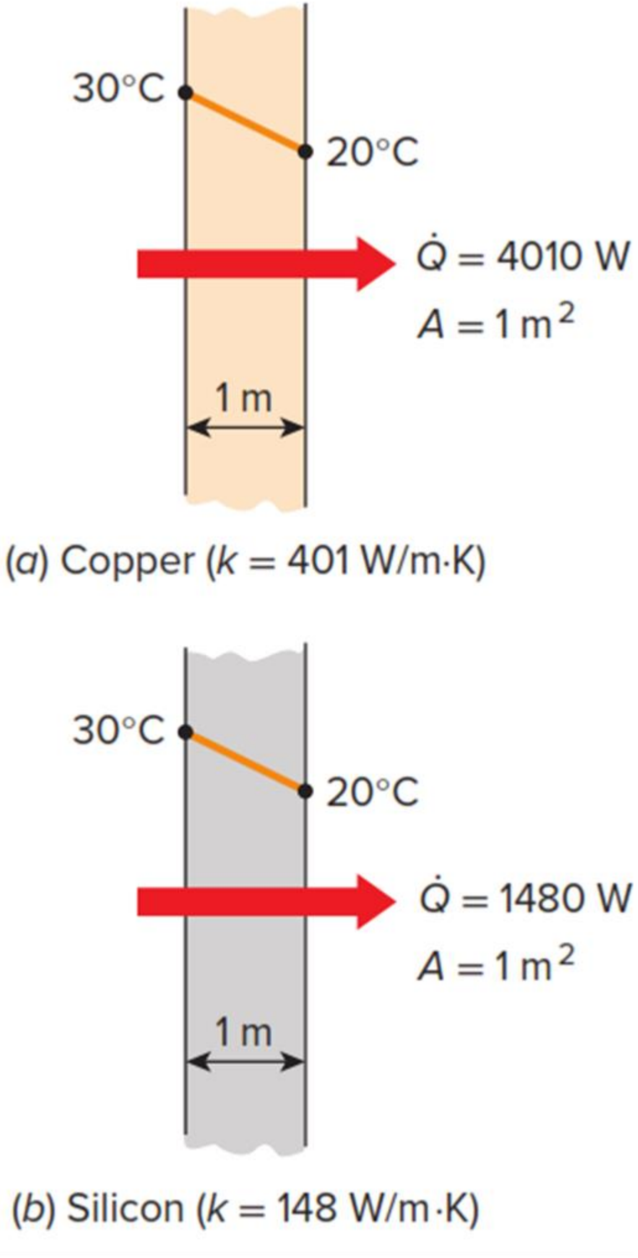
In the limiting case of $\Delta X \rightarrow 0$, the equation above reduces to the differential form:

$$\dot{Q}_{cond} = -kA\frac{dT}{dx} \qquad (W)$$

which is called Fourier's law of heat conduction

dT/dx is the temperature* gradient, which is the slope of the temperature curve on a *T-x diagram (the* rate of change of *T with x), at location x.

The *negative sign in Eq. 16–2 ensures that* heat transfer in the positive *x direction is a positive quantity.*



(a) Copper (k = 401 W/m·K)

(b) Silicon (k = 148 W/m·K)

Fig. 16–2

EXAMPLE 16–1 The Cost of Heat Loss through a Roof

The roof of an electrically heated home is 6 m long, 8 m wide, and 0.25 m thick, and is made of a flat layer of concrete whose thermal conductivity is $k = 0.8$ W/m·K (Fig. 16–4). The temperatures of the inner and the outer surfaces of the roof one night are measured to be 15°C and 4°C, respectively, for a period of 10 hours. Determine (*a*) the rate of heat loss through the roof that night and (*b*) the cost of that heat loss to the home owner if the cost of electricity is \$0.08/kWh.

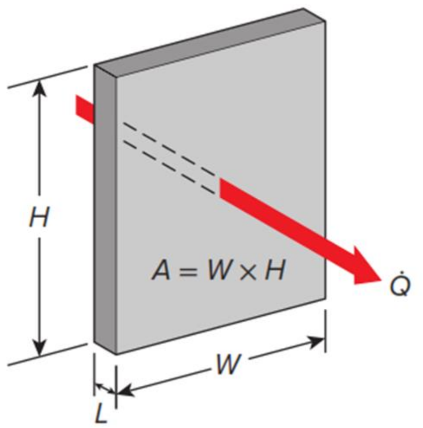


FIGURE 16–3

In heat conduction, *A is* the area *normal to the* direction of heat transfer.

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8\text{ K})(48\text{ m}^2)\frac{(15 - 4)°\text{C}}{0.25\text{ m}} = \mathbf{1690\ W = 1.69\ kW}$$

(*b*) The amount of heat lost through the roof during a 10-hour period and its cost is

$$Q = \dot{Q}\,\Delta t = (1.69\text{ kW})(10\text{ h}) = 16.9\text{ kWh}$$

$$\begin{aligned}\text{Cost} &= (\text{Amount of energy})(\text{Unit cost of energy})\\ &= (16.9\text{ kWh})(\$0.08/\text{kWh}) = \mathbf{\$1.35}\end{aligned}$$

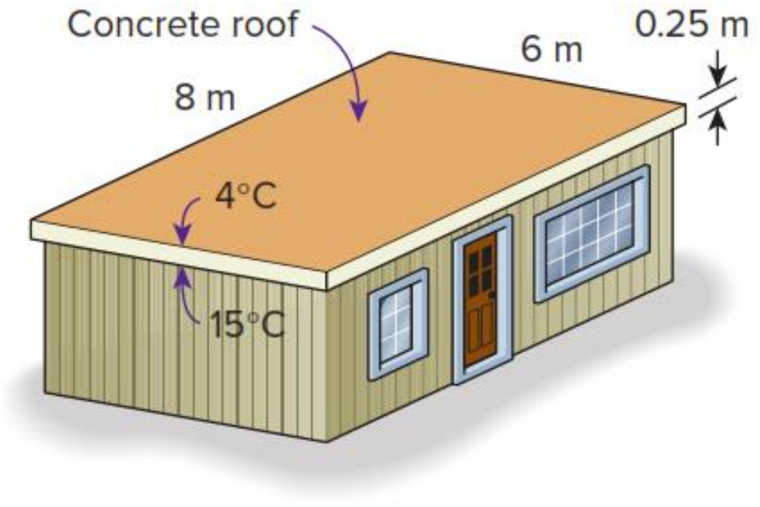


FIGURE 16–4

Schematic for Example 16–1.

Thermal Conductivity

Thermal conductivity *k* *is a measure of a material's ability to conduct* heat.

For example, *k = 0.607 W/m·K for water and k = 80.2 W/m·K for* iron at room temperature, which indicates that iron conducts heat more than 100 times faster than water can.

Water is a poor heat conductor relative to iron.

Thermal conductivity of a material can be defined as *the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.*

A high value for thermal conductivity indicates that the material is a good heat conductor.

a low value indicates that the material is a poor heat conductor or *insulator.*

TABLE 16–1

The thermal conductivities of some materials at room temperature

Material	*k*, W/m·K*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.607
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

*Multiply by 0.5778 to convert to Btu/h·ft·°F.

A layer of material of known thickness and area can be heated from one side by an electric resistance heater of known output.

If the outer surfaces of the heater are well insulated, all the heat generated by the resistance heater will be transferred through the material whose conductivity is to be determined.

Then measuring the two surface temperatures of the material when steady heat transfer is reached and substituting them into Eq. 16–1 together with other known quantities give the thermal conductivity (Fig. 16–5).

Temperature is a measure of the kinetic energies of the particles such as the molecules or atoms of a substance.

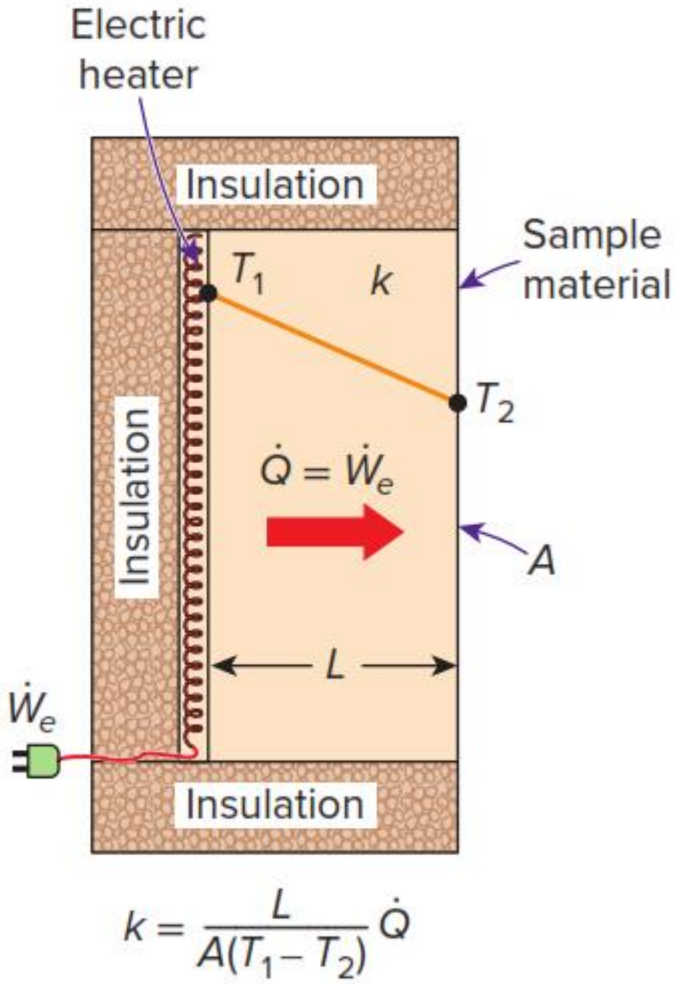
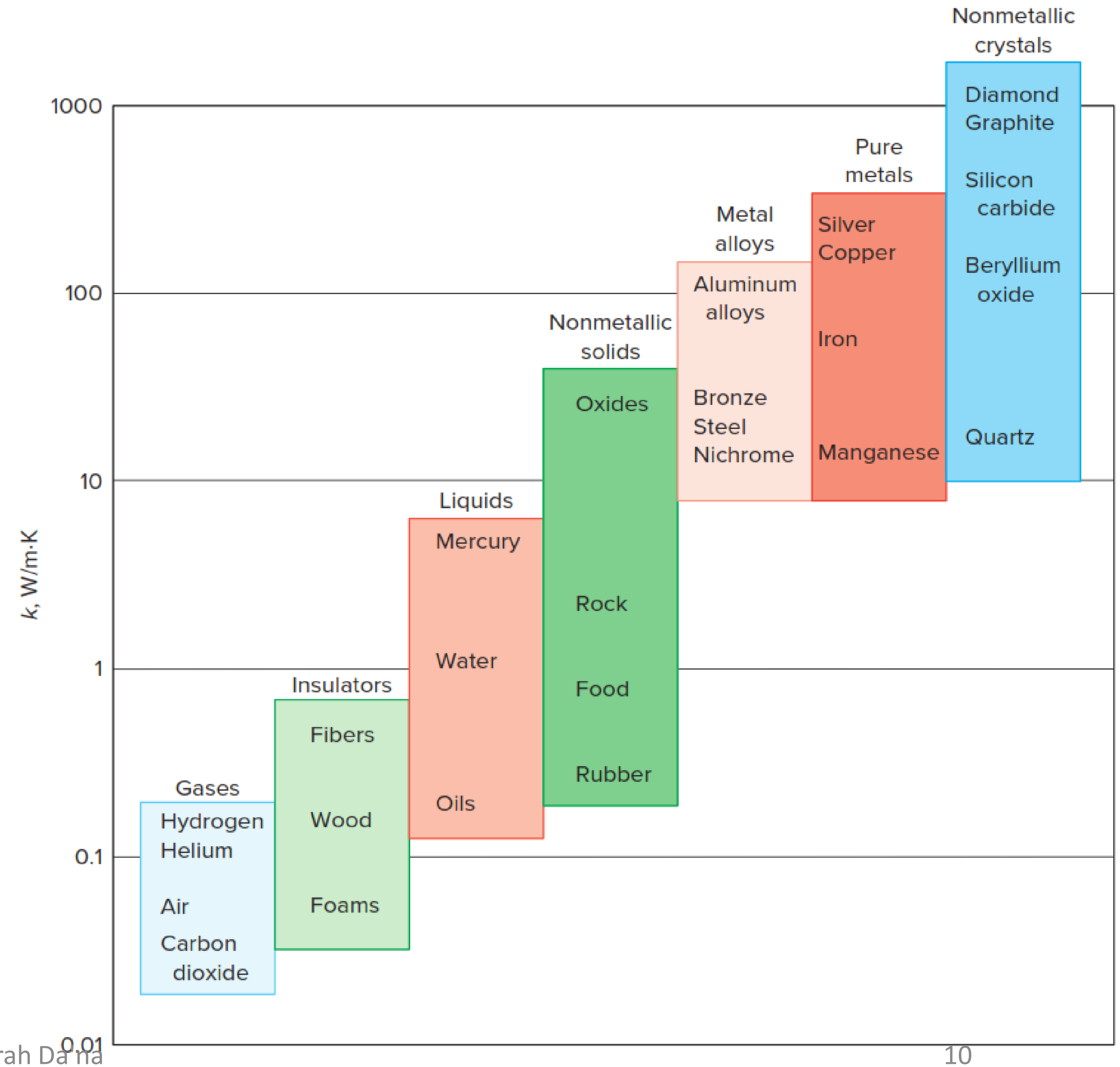


FIGURE 16–5

Thermal conductivity of gases is proportional to the *square root of the thermodynamic temperature T,* and inversely proportional to the *square root of the molar mass M.*

For example, at a fixed temperature of 1000 K, the thermal conductivity of helium (*M* = *4)* is 0.343 W/m·*K and that of air (M = 29) is 0.0667 W/m·K, which is much* lower than that of helium.

The thermal conductivity of gases is *independent of pressure in a wide* range of pressures encountered in practice.



The thermal conductivity of a substance is normally highest in the solid phase and lowest in the gas phase.

The thermal conductivity of liquids is generally insensitive to pressure except near the thermodynamic critical point.

The thermal conductivities of most liquids decrease with increasing temperature, with water being a notable exception.

The conductivity of liquids decreases with increasing molar mass.

Liquid metals such as mercury and sodium have high thermal conductivities and are very suitable for use in applications where a high heat transfer rate to a liquid is desired, as in nuclear power plants.

In *solids, heat conduction is due to two effects: the lattice vibrational waves and flow of free electrons.*

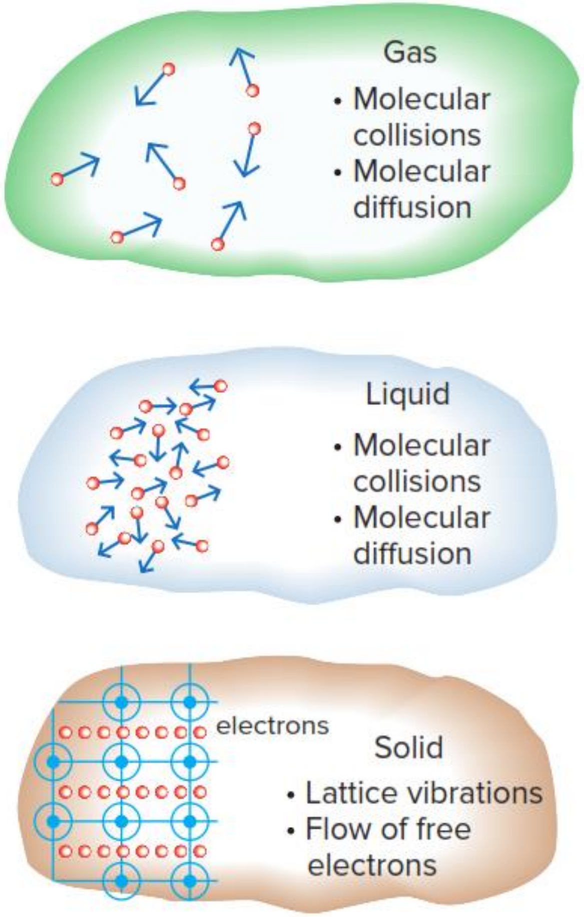


FIGURE 16–7
The mechanisms of heat conduction in different phases of a substance.

***Crystalline solids such as diamond and semiconductors such as silicon are good heat* conductors but poor electrical conductors.**

The thermal conductivity of an alloy of two metals is usually much lower than that of either metal, as shown in Table 16–2.

For example, the thermal conductivity of steel containing just 1 percent of chrome is 62 W/m·K, while the thermal conductivities of iron and chromium are 83 and 95 W/m·K, respectively.

The thermal conductivities of materials vary with temperature (Table 16–3).

TABLE 16–2

The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

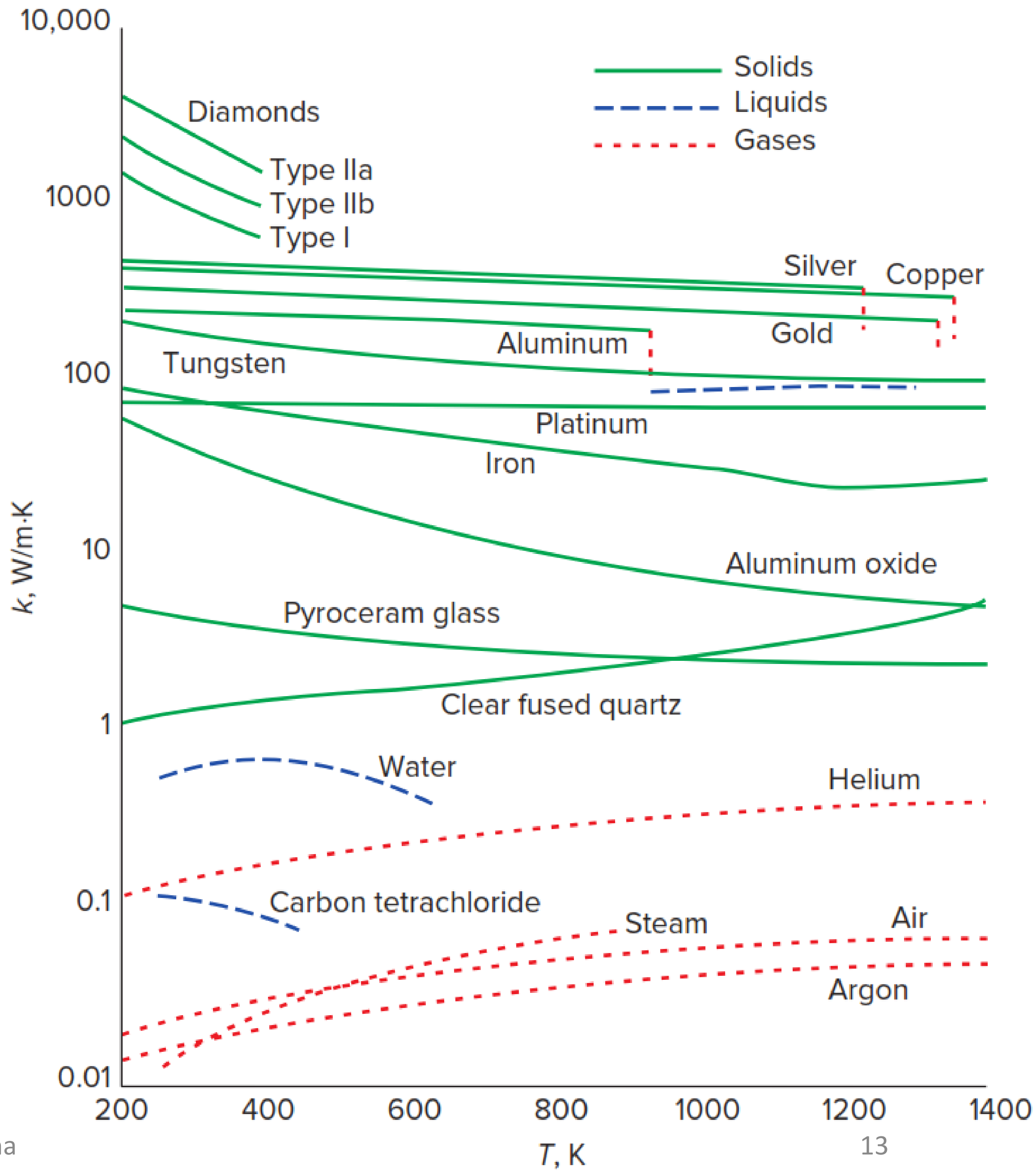
Pure metal or alloy	k, W/m·K, at 300 K
Copper	401
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze (90% Cu, 10% Al)	52

The variation of thermal conductivity over certain temperature ranges is negligible for some materials, but significant or others, as shown in Fig. 16–8.

TABLE 16–3

Thermal conductivities of materials vary with temperature

	k, W/m·K	
T, *K*	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218



The thermal conductivities of certain solids exhibit dramatic increases at temperatures near absolute zero, when these solids become *superconductors*.

The thermal conductivities and other thermal properties of various materials are given in Tables A-15 to A-25.

It is common practice to evaluate the thermal conductivity *k* at the *average temperature* and treat it as a *constant* in calculations.

In heat transfer analysis, a material is normally assumed to be *isotropic;* that is, to have uniform properties in all directions.

Thermal Diffusivity

The product *ρcp* , which is frequently encountered in heat transfer analysis, is called the heat capacity of a material. Both the specific heat *cp* and the heat capacity *ρcp* represent the heat storage capability of a material. But *cp* expresses it *per unit mass whereas ρcp* expresses it *per unit volume*, *as can* be noticed from their units J/kg·K and J/m^3·K, respectively.

Another material property that appears in the transient heat conduction analysis is the thermal diffusivity, which represents how fast heat diffuses through a material and is defined as

$$\alpha = \frac{\text{Heat conduction}}{\text{Heat storage}} = \frac{k}{\rho c_p} \qquad (\text{m}^2/\text{s})$$

Thermal conductivity *k represents how well a material con*ducts heat, and the heat capacity*ρcp* represents how much energy a material stores per unit volume. Therefore, the thermal diffusivity of a material can be viewed as the ratio of the *heat conducted through the material to the heat stored per unit volume.*

TABLE 16–4

The thermal diffusivities of some materials at room temperature

Material	α, m^2/s*
Silver	149×10^{-6}
Gold	127×10^{-6}
Copper	113×10^{-6}
Aluminum	97.5×10^{-6}
Iron	22.8×10^{-6}
Mercury (l)	4.7×10^{-6}
Marble	1.2×10^{-6}
Ice	1.2×10^{-6}
Concrete	0.75×10^{-6}
Brick	0.52×10^{-6}
Heavy soil (dry)	0.52×10^{-6}
Glass	0.34×10^{-6}
Glass wool	0.23×10^{-6}
Water (l)	0.14×10^{-6}
Beef	0.14×10^{-6}
Wood (oak)	0.13×10^{-6}

*Multiply by 10.76 to convert to ft^2/s.

16-3 CONVECTION

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of *conduction and fluid motion.*

The faster the fluid motion, the greater the convection heat transfer.

In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.

Consider the cooling of a hot block by blowing cool air over its top surface (Fig. 16–11).

Heat is first transferred to the air layer adjacent to the block by conduction. This heat is then carried away from the surface by convection, that is, by the combined effects of conduction within the air that is due to random motion of air molecules and the bulk or macroscopic motion of the air that removes the heated air near the surface and replaces it by the cooler air.

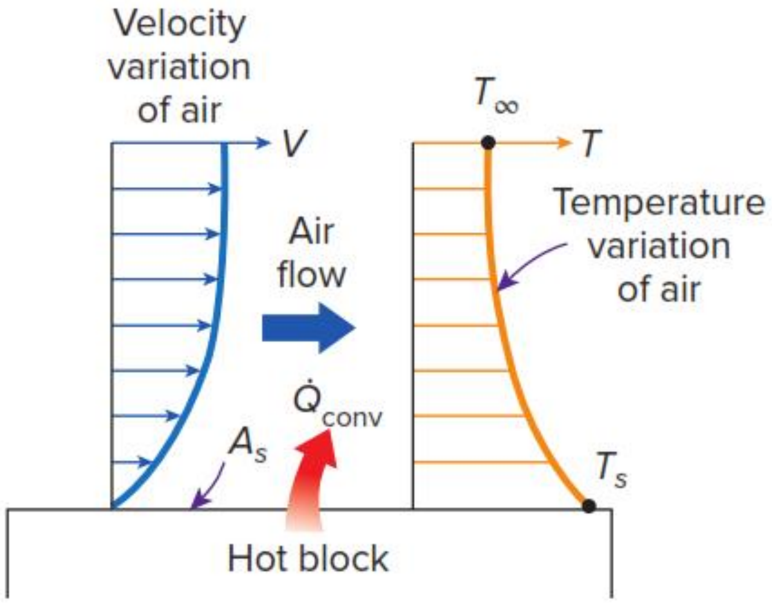


FIGURE 16–11
Heat transfer from a hot surface to air by convection.

Convection is called forced convection if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind.

convection is called natural (or free) convection if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid (Fig. 16–12).

The rate of *convection heat transfer* is observed to be proportional to the temperature difference, and is conveniently expressed by Newton's law of cooling as:

$$\dot{Q}_{conv} = hA_s (T_s - T_\infty) \qquad \text{(W)}$$

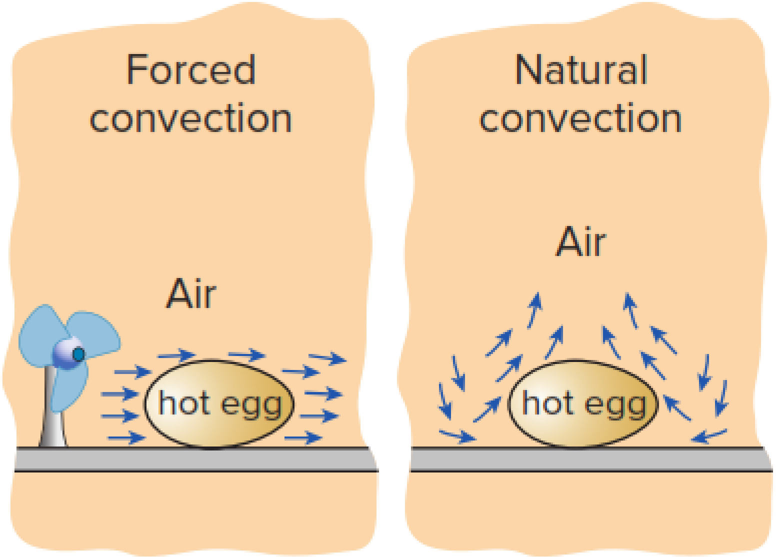


FIGURE 16–12
The cooling of a boiled egg by forced and natural convection.

where *h is the convection heat transfer coefficient in W/m*^{2}·K or Btu/h·ft·°F, *As* is the surface area through which convection heat transfer takes place, *Ts* is the surface temperature, and *T*∞ is the temperature of the fluid sufficiently far from the surface.

Note that at the surface, the fluid temperature equals the surface temperature of the solid.

The convection heat transfer coefficient *h is not a property of the fluid. It* is an experimentally determined parameter whose value depends on all the variables influencing convection such as the surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity. Typical values of *h are given in Table 16–5.*

TABLE 16–5

Typical values of convection heat transfer coefficient

Type of convection	*h*, W/m^2·K*
Free convection of gases	2–25
Free convection of liquids	10–1000
Forced convection of gases	25–250
Forced convection of liquids	50–20,000
Boiling and condensation	2500–100,000

*Multiply by 0.176 to convert to Btu/h·ft^2·°F.

EXAMPLE 16–4 Measuring Convection Heat Transfer Coefficient

A 2-m-long, 0.3-cm-diameter electrical wire extends across a room at 15°C, as shown in Fig. 16–13. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 60 V and 1.5 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

Assumptions **1** Steady operating conditions exist since the temperature readings do not change with time. **2** Radiation heat transfer is negligible.

$$\dot{Q} = \dot{E}_{\text{generated}} = \mathbf{V}I = (60\text{ V})(1.5\text{ A}) = 90\text{ W}$$

The surface area of the wire is

$$A_s = \pi DL = \pi(0.003\text{ m})(2\text{ m}) = 0.01885\text{ m}^2$$

Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q}_{\text{conv}} = hA_s\,(T_s - T_\infty)$$

$$h = \frac{\dot{Q}_{\text{conv}}}{A_s(T_s - T_\infty)} = \frac{90\text{ W}}{(0.01885\text{ m}^2)(152 - 15)°\text{C}} = \mathbf{34.9\text{ W/m}^2\cdot\text{K}}$$

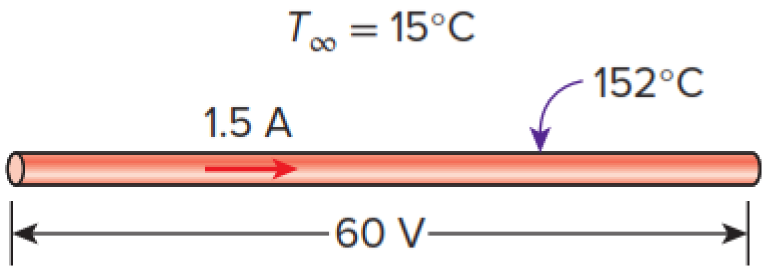


FIGURE 16–13
Schematic for Example 16–4.

16-4 RADIATION

Radiation is the energy emitted by matter in the form of *electromagnetic waves (or photons) as a result of the changes in the electronic configurations* of the atoms or molecules.

Heat transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum.

All bodies at a temperature above absolute zero emit thermal radiation.

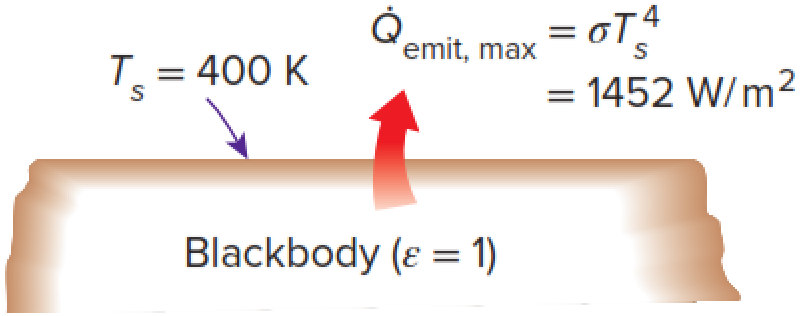
Radiation is a *volumetric phenomenon, and all solids, liquids, and gases* emit, absorb, or transmit radiation to varying degrees. However, radiation is usually considered to be a *surface phenomenon for solids that are opaque* to thermal radiation such as metals, wood, and rocks since the radiation emitted by the interior regions of such material can never reach the surface, and the radiation incident on such bodies is usually absorbed within a few micrometers from the surface.

The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature *T* (in K or R) is given by the Stefan–Boltzmann law as:

$$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4 \qquad \text{(W)}$$

where $\sigma = 5.670 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$ or $0.1714 \times 10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4$ is the *Stefan–Boltzmann constant*. The idealized surface that emits radiation at this maximum rate is called a **blackbody,** and the radiation emitted by a blackbody is called **blackbody radiation** (Fig. 16–14). The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$\dot{Q}_{\text{emit}} = \varepsilon \sigma A_s T_s^4 \qquad \text{(W)}$$



where ε is the emissivity of the surface. The property emissivity, whose value is in the range 0≤ε ≤ 1, is a measure of how closely a surface approximates a blackbody for which ε = 1. The emissivities of some surfaces are given in Table 16–6.

Another important radiation property of a surface is its absorptivity a, which is the fraction of the radiation energy incident on a surface that is absorbed by the surface. Like emissivity, its value is in the range 0 ≤ α ≤ 1. A blackbody absorbs the entire radiation incident on it. That is, a blackbody is a perfect absorber (α = 1) as it is a perfect emitter.

TABLE 16–6

Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

In general, both ε and a of a surface depend on the temperature and the wavelength of the radiation.

Kirchhoff's law of radiation states that the emissivity and the absorptivity of a surface at a given temperature and wavelength are equal.

In many practical applications, the surface temperature and the temperature of the source of incident radiation are of the same order of magnitude, and the average absorptivity of a surface is taken to be equal to its average emissivity.

The rate at which a surface absorbs radiation is determined from (Fig. 16–15).

$$\dot{Q}_{\text{absorbed}} = \alpha \dot{Q}_{\text{incident}} \qquad \text{(W)}$$

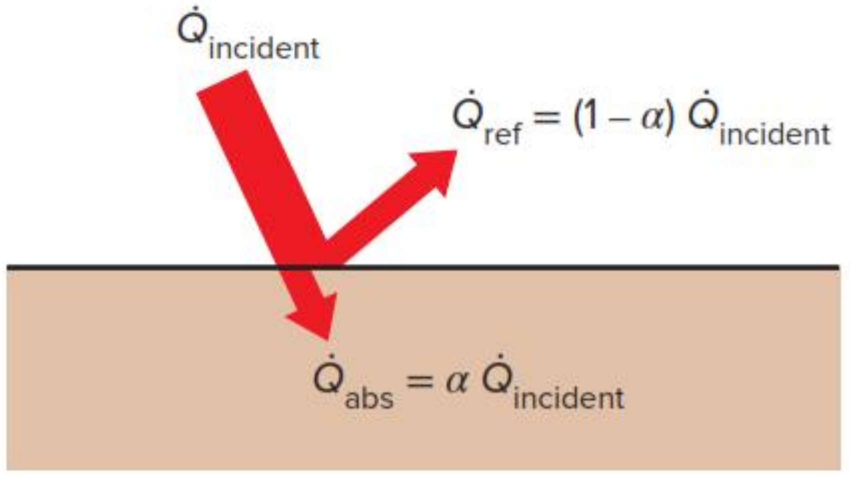


FIGURE 16–15
The absorption of radiation incident on an opaque surface of absorptivity α.

where *Q* Incident is the rate at which radiation is incident on the surface and a is the absorptivity of the surface. For opaque (nontransparent) surfaces, the portion of incident radiation not absorbed by the surface is reflected back.

The difference between the rates of radiation emitted by the surface and the radiation absorbed is the *net radiation heat transfer.*

If the rate of radiation absorption is greater than the rate of radiation emission, the surface is said to be *gaining* energy by radiation.

Otherwise, the surface is said to be *losing energy by radiation.*

When a surface of emissivity ε and surface area *As* at a *thermodynamic temperature Ts* is *completely enclosed by a much larger (or black) surface at* thermodynamic temperature *T*surr separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by (Fig. 16–16)

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s\,(T_s^4 - T_{\text{surr}}^4) \qquad \text{(W)}$$

Radiation heat transfer to or from a surface surrounded by a gas such as air occurs *parallel to conduction (or convection, if there is bulk gas motion)* between the surface and the gas.

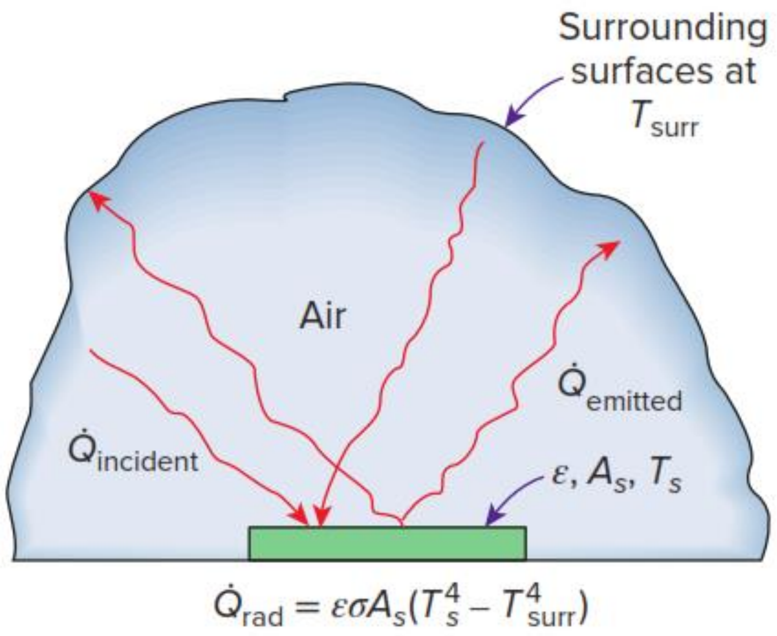


FIGURE 16–16

Thus the total heat transfer is determined by *adding the contributions of both heat transfer mechanisms. For simplicity* and convenience, this is often done by defining a **combined heat transfer coefficient** *h* that includes the effects of both convection and radiation. Then the *total heat transfer rate to or from a surface by convection and radiation* is expressed as:

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = h_{\text{conv}} A_s (T_s - T_{\text{surr}}) + \varepsilon\sigma A_s (T_s^4 - T_{\text{surr}}^4)$$

$$\dot{Q}_{\text{total}} = h_{\text{combined}} A_s (T_s - T_\infty) \qquad \text{(W)}$$

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} = h_{\text{conv}} + \varepsilon\sigma (T_s + T_{\text{surr}})(T_s^2 + T_{\text{surr}}^2)$$

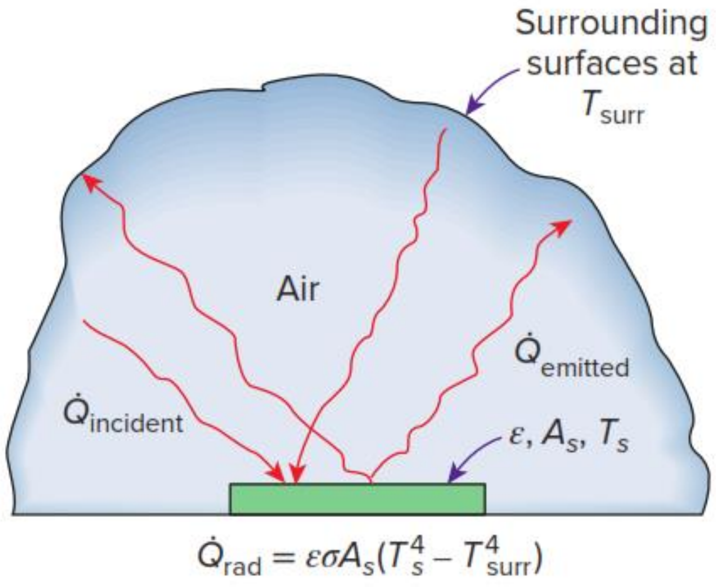


FIGURE 16–16

EXAMPLE 16–5 Radiation Effect on Thermal Comfort

It is a common experience to feel "chilly" in winter and "warm" in summer in our homes even when the thermostat setting is kept the same. This is due to the so-called "radiation effect" resulting from radiation heat exchange between our bodies and the surrounding surfaces of the walls and the ceiling.

Consider a person standing in a room maintained at 22°C at all times. The inner surfaces of the walls, floors, and the ceiling of the house are observed to be at an average temperature of 10°C in winter and 25°C in summer. Determine the rate of radiation heat transfer between this person and the surrounding surfaces if the exposed surface area and the average outer surface temperature of the person are 1.4 m^2 and 30°C, respectively (Fig. 16–17).

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by convection is not considered. **3** The person is completely surrounded by the interior surfaces of the room. **4** The surrounding surfaces are at a uniform temperature.

Properties The emissivity of a person is $\varepsilon = 0.95$ (Table 16–6).

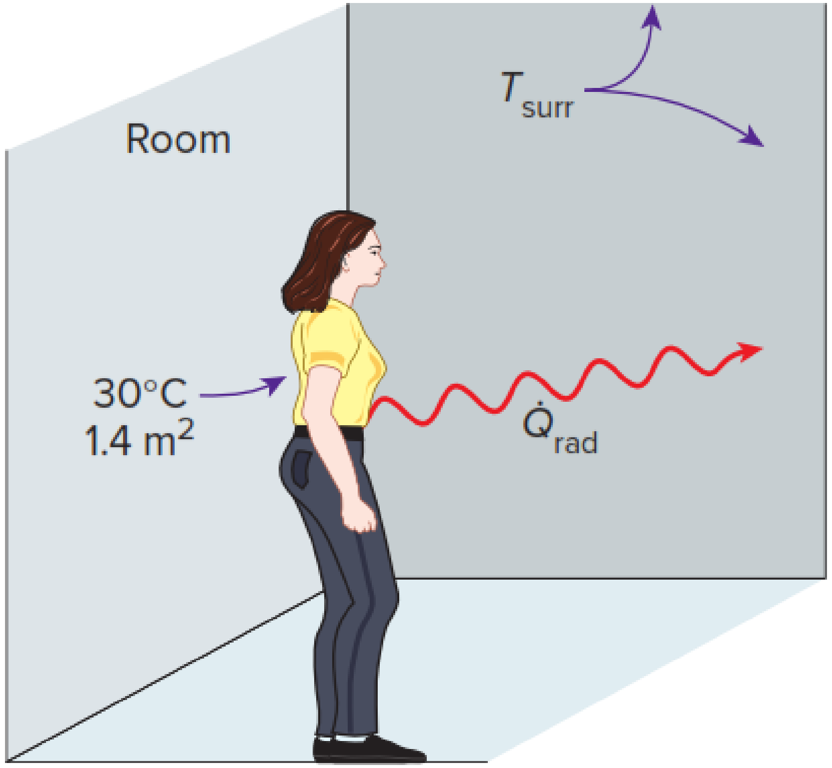


FIGURE 16–17

Schematic for Example 16–5.

Analysis The net rates of radiation heat transfer from the body to the surrounding walls, ceiling, and floor in winter and summer are

$$
\begin{aligned}
\dot{Q}_{\text{rad, winter}} &= \varepsilon\sigma A_s (T_s^4 - T_{\text{surr, winter}}^4) \\
&= (0.95)(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.4\ \text{m}^2) \\
&\quad \times [(30 + 273)^4 - (10 + 273)^4]\ \text{K}^4 \\
&= \mathbf{152\ W}
\end{aligned}
$$

$$
\begin{aligned}
\dot{Q}_{\text{rad, summer}} &= \varepsilon\sigma A_s (T_s^4 - T_{\text{surr, summer}}^4) \\
&= (0.95)(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.4\ \text{m}^2) \\
&\quad \times [(30 + 273)^4 - (25 + 273)^4]\ \text{K}^4 \\
&= \mathbf{40.9\ W}
\end{aligned}
$$

16-5 SIMULTANEOUS HEAT TRANSFER MECHANISMS

Heat transfer is only by conduction in *opaque solids*.

But by conduction and radiation in *semitransparent solids*.

Thus, a solid may involve conduction and radiation but not convection.

A solid may involve heat transfer by convection and/or radiation on its surfaces exposed to a fluid or other surfaces.

Heat transfer is by conduction and possibly by radiation in a *still fluid (no* bulk fluid motion) and by convection and radiation in a *flowing fluid*.

***In the* absence of radiation, heat transfer through a fluid is either by conduction or convection, depending on the presence of any bulk fluid motion.**

Convection can be viewed as combined conduction and fluid motion, and conduction in a fluid can be viewed as a special case of convection in the absence of any fluid motion (Fig. 16–18).

Thus, when we deal with heat transfer through a *fluid, we have either conduction or convection, but not both.*

heat transfer through a *vacuum is by radiation only since conduction* or convection requires the presence of a material medium.

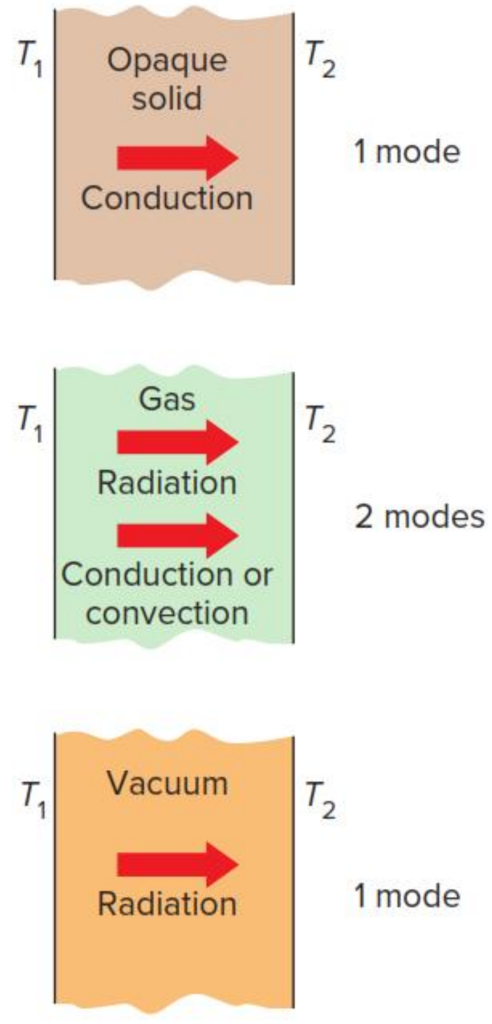


FIGURE 16–18

EXAMPLE 16–6 Heat Loss from a Person

Consider a person standing in a breezy room at 20°C. Determine the total rate of heat transfer from this person if the exposed surface area and the average outer surface temperature of the person are 1.6 m^2 and 29°C, respectively, and the convection heat transfer coefficient is 6 $W/m^2 \cdot K$ (Fig. 16–19).

Assumptions **1** Steady operating conditions exist. **2** The person is completely surrounded by the interior surfaces of the room. **3** The surrounding surfaces are at the same temperature as the air in the room. **4** Heat conduction to the floor through the feet is negligible.

Properties The emissivity of a person is $\varepsilon = 0.95$ (Table 16–6).

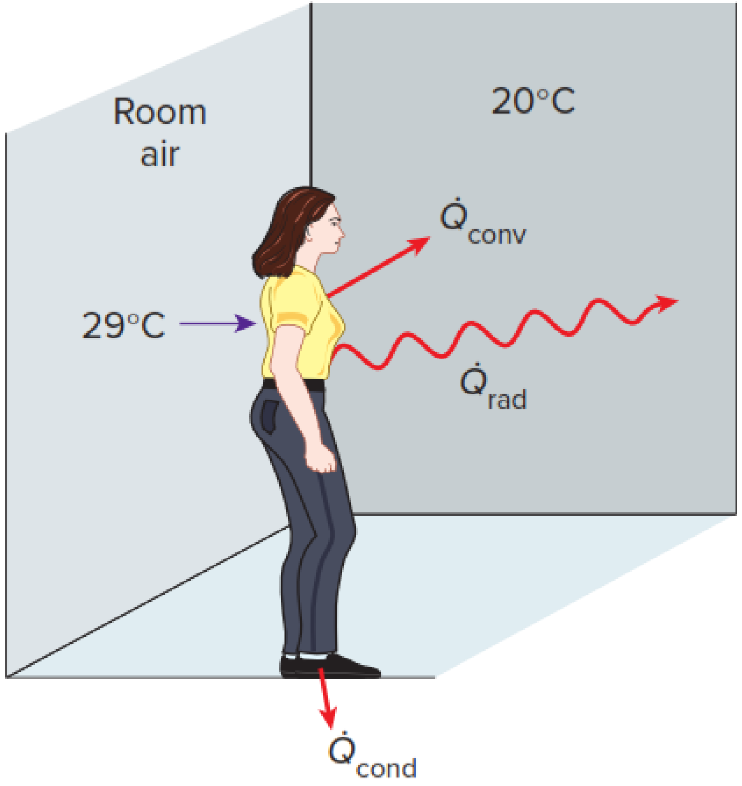


FIGURE 16–19
Heat transfer from the person described in Example 16–6.

Analysis The heat transfer between the person and the air in the room is by convection (instead of conduction) since it is conceivable that the air in the vicinity of the skin or clothing warms up and rises as a result of heat transfer from the body, initiating natural convection currents. It appears that the experimentally determined value for the rate of convection heat transfer in this case is 6 W per unit surface area (m^2) per unit temperature difference (in K or °C) between the person and the air away from the person. Thus, the rate of convection heat transfer from the person to the air in the room is

$$\begin{aligned}\dot{Q}_{\text{conv}} &= hA_s(T_s - T_\infty)\\ &= (6\ \text{W/m}^2\cdot\text{K})(1.6\ \text{m}^2)(29 - 20)^\circ\text{C}\\ &= 86.4\ \text{W}\end{aligned}$$

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)\\ &= (0.95)(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.6\ \text{m}^2)\\ &\quad \times [(29 + 273)^4 - (20 + 273)^4]\ \text{K}^4\\ &= 81.7\ \text{W}\end{aligned}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = (86.4 + 81.7)\ \text{W} \cong \mathbf{168\ W}$$

EXAMPLE 16–7 Heat Transfer between Two Isothermal Plates

Consider steady heat transfer between two large parallel plates at constant temperatures of $T_1 = 300$ K and $T_2 = 200$ K that are $L = 1$ cm apart, as shown in Fig. 16–20. Assuming the surfaces to be black (emissivity $\varepsilon = 1$), determine the rate of heat transfer between the plates per unit surface area assuming the gap between the plates is (*a*) filled with atmospheric air, (*b*) evacuated, (*c*) filled with urethane insulation, and (*d*) filled with superinsulation that has an apparent thermal conductivity of 0.00002 W/m·K.

Assumptions **1** Steady operating conditions exist. **2** There are no natural convection currents in the air between the plates. **3** The surfaces are black and thus $\varepsilon = 1$.

Properties The thermal conductivity at the average temperature of 250 K is $k = 0.0219$ W/m·K for air (Table A–22), 0.026 W/m·K for urethane insulation, and 0.00002 W/m·K for the superinsulation.

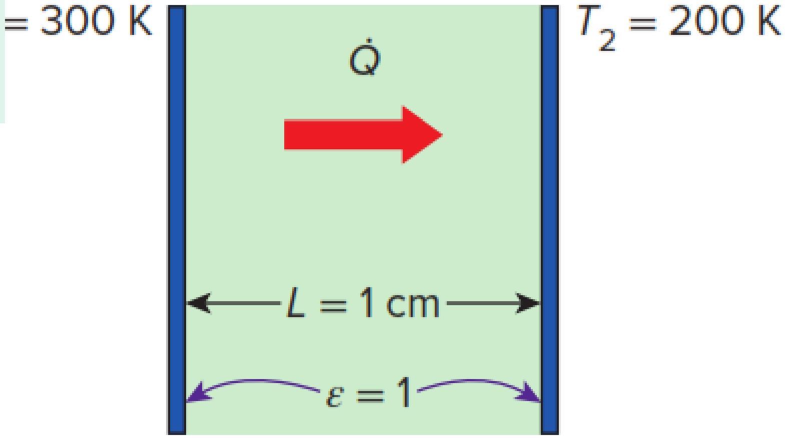


FIGURE 16–20
Schematic for Example 1–11.

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.0219\ \text{W/m}\cdot\text{K})(1\ \text{m}^2)\frac{(310 - 200)\text{K}}{0.01\ \text{m}} = 219\ \text{W}$$

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma \text{A}(T_1^4 - T_2^4)$$

$$= (1)(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1\ \text{m}^2)[(300\ \text{K})^4 - (200\ \text{K})^4] = 369\ \text{W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} + \dot{Q}_{\text{rad}} = 219 + 369 = \mathbf{588\ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{rad}} = \mathbf{369\ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.026\ \text{W/m}\cdot\text{K})(1\ \text{m}^2)\frac{(300 - 200)\text{K}}{0.01\ \text{m}} = \mathbf{260\ W}$$

$$\dot{Q}_{\text{total}} = kA\frac{T_1 - T_2}{L} = (0.00002\ \text{W/m}\cdot\text{K})(1\ \text{m}^2)\frac{(300 - 200)\text{K}}{0.01\ \text{m}} = \mathbf{0.2\ W}$$

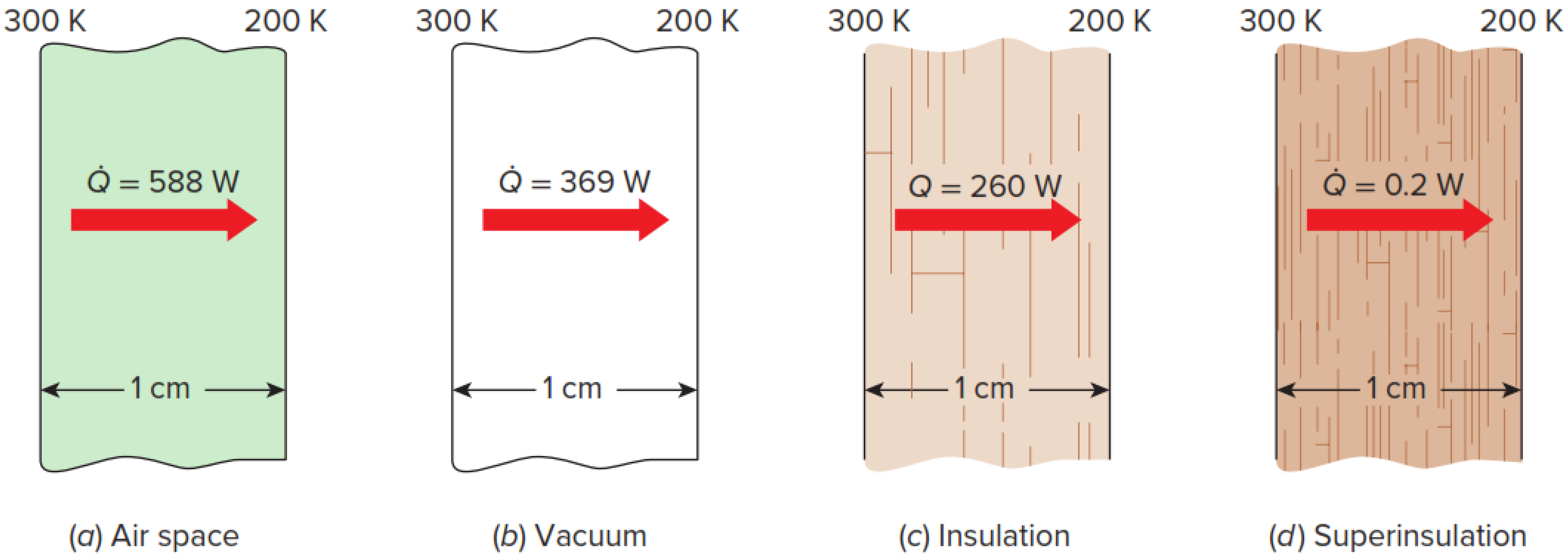


FIGURE 16–21
Different ways of reducing heat transfer between two isothermal plates, and their effectiveness.

EXAMPLE 16–9 Heating of a Plate by Solar Energy

A thin metal plate is insulated on the back and exposed to solar radiation at the front surface (Fig. 16–23). The exposed surface of the plate has an absorptivity of 0.6 for solar radiation. If solar radiation is incident on the plate at a rate of 700 W/m^2 and the surrounding air temperature is 25°C, determine the surface temperature of the plate when the heat loss by convection and radiation equals the solar energy absorbed by the plate. Assume the combined convection and radiation heat transfer coefficient to be 50 W/m^2·K.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulated side of the plate is negligible. **3** The heat transfer coefficient remains constant.

Properties The solar absorptivity of the plate is given to be $\alpha = 0.6$.

$$\dot{E}_{\text{gained}} = \dot{E}_{\text{lost}} \quad \text{or} \quad \alpha A_s \dot{q}_{\text{incident, solar}} = h_{\text{combined}} A_s (T_s - T_\infty)$$

Solving for T_s and substituting, the plate surface temperature is determined to be

$$T_s = T_\infty + \alpha \frac{\dot{q}_{\text{incident, solar}}}{h_{\text{combined}}} = 25°\text{C} + \frac{0.6 \times (700\ \text{W/m}^2)}{50\ \text{W/m}^2\cdot\text{K}} = 33.4°\text{C}$$

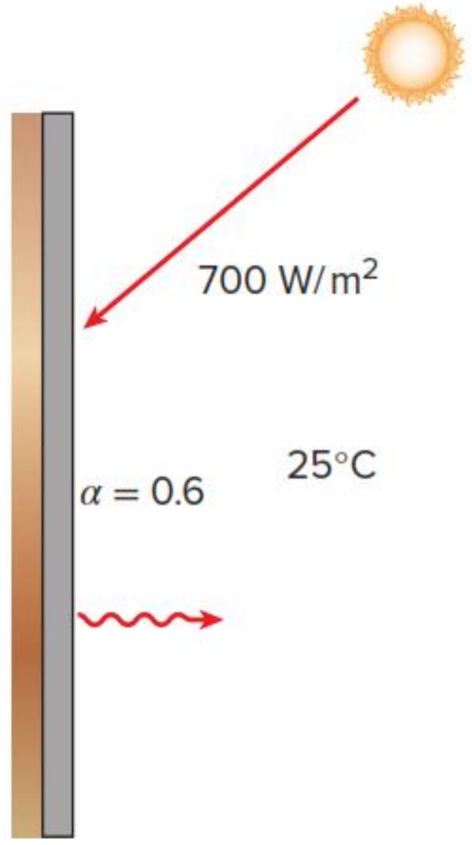


FIGURE 16–23
Schematic for Example 16–9.

SUMMARY

Heat can be transferred in three different modes: conduction, convection, and radiation. *Conduction* is the transfer of heat from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles, and is expressed by *Fourier's law of heat conduction* as

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx}$$

where k is the *thermal conductivity* of the material, A is the *area* normal to the direction of heat transfer, and dT/dx is the *temperature gradient*. The magnitude of the rate of heat conduction across a plane layer of thickness L is given by

$$\dot{Q}_{\text{cond}} = kA\frac{\Delta T}{L}$$

where ΔT is the temperature difference across the layer.

Convection is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion, and involves the combined effects of conduction and fluid motion. The rate of convection heat transfer is expressed by *Newton's law of cooling as*

$$\dot{Q}_{\text{convection}} = hA_s(T_s - T_\infty)$$

where h is the *convection heat transfer coefficient* in W/m^2·K or Btu/h·ft^2·R, A_s is the *surface area* through which convection heat transfer takes place, T_s is the *surface temperature*, and T_∞ is the *temperature of the fluid* sufficiently far from the surface.

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s is given by the *Stefan–Boltzmann law* as $\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4$, where $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4 or 0.1714×10^{-8} Btu/h·ft^2·R^4 is the *Stefan–Boltzmann constant*.

When a surface of emissivity ε and surface area A_s at a temperature T_s is completely enclosed by a much larger (or black) surface at a temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)$$

In this case, the emissivity and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

The rate at which a surface absorbs radiation is determined from $\dot{Q}_{\text{absorbed}} = \alpha\dot{Q}_{\text{incident}}$ where $\dot{Q}_{\text{incident}}$ is the rate at which radiation is incident on the surface and α is the absorptivity of the surface.