25.1 CURRENT

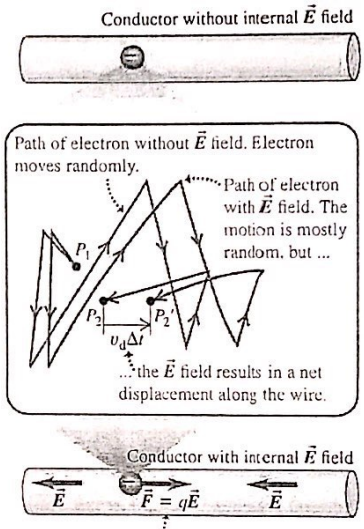
A **current** is any motion of charge from one region to another. In this section we'll discuss currents in conducting materials. The vast majority of technological applications of charges in motion involve currents of this kind.

In electrostatic situations (discussed in Chapters 21 through 24) the electric field is zero everywhere within the conductor, and there is *no* current. However, this does not mean that all charges within the conductor are at rest. In an ordinary metal such as copper or aluminum, some of the electrons are free to move within the conducting material. These free electrons move randomly in all directions, somewhat like the molecules of a gas but with much greater speeds, of the order of 10^6 m/s. The electrons nonetheless do not escape from the conducting material, because they are attracted to the positive ions of the material. The motion of the electrons is random, so there is no *net* flow of charge in any direction and hence no current.

Now consider what happens if a constant, steady electric field $\vec{E}$ is established inside a conductor. (We'll see later how this can be done.) A charged particle (such as a free electron) inside the conducting material is then subjected to a steady force $\vec{F} = q\vec{E}$. If the charged particle were moving in *vacuum*, this steady force would cause a steady acceleration in the direction of $\vec{F}$, and after a time the charged particle would be moving in that direction at high speed. But a charged particle moving in a *conductor* undergoes frequent collisions with the massive, nearly stationary ions of the material. In each such collision the particle's direction of motion undergoes a random change. The net effect of the electric field $\vec{E}$ is that in addition to the random motion of the charged particles within the conductor, there is also a very slow net motion or *drift* of the moving charged particles as a group in the direction of the electric force $\vec{F} = q\vec{E}$ (**Fig. 25.1**). This motion is described in terms of the **drift velocity** $\vec{v}_d$ of the particles. As a result, there is a net current in the conductor.

While the random motion of the electrons has a very fast average speed of about 10^6 m/s, the drift speed is very slow, often on the order of 10^{-4} m/s. Given that the electrons move so slowly, you may wonder why the light comes on immediately when you turn on the switch of a flashlight. The reason is that the electric field is set up in the wire with a speed approaching the speed of light, and electrons start to move all along the wire at very nearly the same time. The time that it takes any individual electron to get from the switch to the light bulb isn't really relevant. A good analogy is a group of soldiers standing at attention when the sergeant orders them to start marching; the order reaches the soldiers' ears at the speed of sound, which is much faster than their marching speed, so all the soldiers start to march essentially in unison.

25.1 If there is no electric field inside a conductor, an electron moves randomly from point P_1 to point P_2 in a time Δt. If an electric field $\vec{E}$ is present, the electric force $\vec{F} = q\vec{E}$ imposes a small drift (greatly exaggerated here) that takes the electron to point P_2', a distance $v_d\Delta t$ from P_2 in the direction of the force.



An electron has a negative charge q, so the force on it due to the $\vec{E}$ field is in the direction opposite to $\vec{E}$.

The Direction of Current Flow

The drift of moving charges through a conductor can be interpreted in terms of work and energy. The electric field $\vec{E}$ does work on the moving charges. The resulting kinetic energy is transferred to the material of the conductor by means of collisions with the ions, which vibrate about their equilibrium positions in the crystalline structure of the conductor. This energy transfer increases the average vibrational energy of the ions and therefore the temperature of the material. Thus much of the work done by the electric field goes into heating the conductor, *not* into making the moving charges move ever faster and faster. This heating is sometimes useful, as in an electric toaster, but in many situations is simply an unavoidable by-product of current flow.

In different current-carrying materials, the charges of the moving particles may be positive or negative. In metals the moving charges are always (negative) electrons, while in an ionized gas (plasma) or an ionic solution the moving charges may include both electrons and positively charged ions. In a semiconductor

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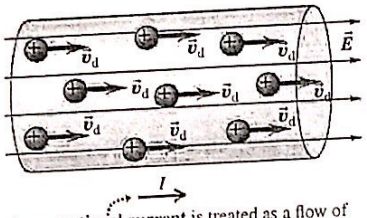
material such as germanium or silicon, conduction is partly by electrons and partly by motion of *vacancies*, also known as *holes*; these are sites of missing electrons and act like positive charges.

Figure 25.2 shows segments of two different current-carrying materials. In Fig. 25.2a the moving charges are positive, the electric force is in the same direction as $\vec{E}$, and the drift velocity $\vec{v}_d$ is from left to right. In Fig. 25.2b the charges are negative, the electric force is opposite to $\vec{E}$, and the drift velocity $\vec{v}_d$ is from right to left. In both cases there is a net flow of positive charge from left to right, and positive charges end up to the right of negative ones. We *define* the current, denoted by I, to be in the direction in which there is a flow of *positive* charge. Thus we describe currents as though they consisted entirely of positive charge flow, even in cases in which we know that the actual current is due to electrons. Hence the current is to the right in both Figs. 25.2a and 25.2b. This choice or convention for the direction of current flow is called **conventional current.** While the direction of the conventional current is *not* necessarily the same as the direction in which charged particles are actually moving, we'll find that the sign of the moving charges is of little importance in analyzing electric circuits.

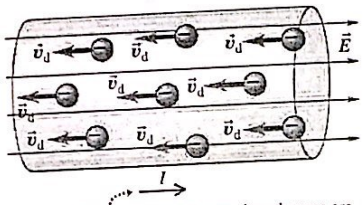
25.2 The same current is produced by (a) positive charges moving in the direction of the electric field $\vec{E}$ or (b) the same number of negative charges moving at the same speed in the direction opposite to $\vec{E}$.

(a)



A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

(b)



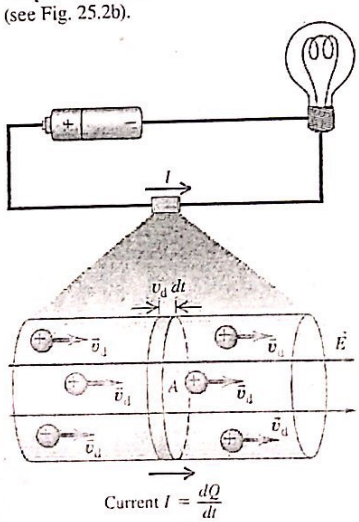
In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

Figure 25.3 shows a segment of a conductor in which a current is flowing. We consider the moving charges to be *positive*, so they are moving in the same direction as the current. We define the current through the cross-sectional area A to be *the net charge flowing through the area per unit time*. Thus, if a net charge dQ flows through an area in a time dt, the current I through the area is

$$I = \frac{dQ}{dt} \quad \text{(definition of current)} \tag{25.1}$$

CAUTION **Current is not a vector** Although we refer to the *direction* of a current, current as defined by Eq. (25.1) is *not* a vector quantity. In a current-carrying wire, the current is always along the length of the wire, regardless of whether the wire is straight or curved. No single vector could describe motion along a curved path. We'll usually describe the direction of current either in words (as in "the current flows clockwise around the circuit") or by choosing a current to be positive if it flows in one direction along a conductor and negative if it flows in the other direction. ℹ

25.3 The current I is the time rate of charge transfer through the cross-sectional area A. The random component of each moving charged particle's motion averages to zero, and the current is in the same direction as $\vec{E}$ whether the moving charges are positive (as shown here) or negative (see Fig. 25.2b).



The SI unit of current is the **ampere**; one ampere is defined to be *one coulomb per second* (1 A = 1 C/s). This unit is named in honor of the French scientist André Marie Ampère (1775–1836). When an ordinary flashlight (D-cell size) is turned on, the current in the flashlight is about 0.5 to 1 A; the current in the wires of a car engine's starter motor is around 200 A. Currents in radio and television circuits are usually expressed in *milliamperes* (1 mA = 10^{-3} A) or *microamperes* (1 μA = 10^{-6} A), and currents in computer circuits are expressed in *nanoamperes* (1 nA = 10^{-9} A) or *picoamperes* (1 pA = 10^{-12} A).

Current, Drift Velocity, and Current Density

We can express current in terms of the drift velocity of the moving charges. Let's reconsider the situation of Fig. 25.3 of a conductor with cross-sectional area A and an electric field $\vec{E}$ directed from left to right. To begin with, we'll assume that the free charges in the conductor are positive; then the drift velocity is in the same direction as the field.

Suppose there are n moving charged particles per unit volume. We call n the concentration of particles; its SI unit is m^{-3}. Assume that all the particles move with the same drift velocity with magnitude v_d. In a time interval dt, each particle moves a distance $v_d\,dt$. The particles that flow out of the right end of the shaded cylinder with length $v_d\,dt$ during dt are the particles that were within this cylinder at the beginning of the interval dt. The volume of the cylinder is $Av_d\,dt$,

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and the number of particles within it is $nAv_d\,dt$. If each particle has a charge q, the charge dQ that flows out of the end of the cylinder during time dt is

$$dQ = q(nAv_d\,dt) = nqv_dA\,dt$$

and the current is

$$I = \frac{dQ}{dt} = nqv_dA$$

The current *per unit cross-sectional area* is called the **current density** J:

$$J = \frac{I}{A} = nqv_d$$

The units of current density are amperes per square meter (A/m^2).

If the moving charges are negative rather than positive, as in Fig. 25.2b, the drift velocity is opposite to $\vec{E}$. But the *current* is still in the same direction as $\vec{E}$ at each point in the conductor. Hence current I and current density J don't depend on the sign of the charge, and so we replace the charge q by its absolute value $|q|$:

Current through an area: $I = \dfrac{dQ}{dt} = n|q|v_dA$ (25.2)

Rate at which charge flows through area; Drift speed; Cross-sectional area; Concentration of moving charged particles; Charge per particle

$$J = \frac{I}{A} = n|q|v_d \quad \text{(current density)} \tag{25.3}$$

The current in a conductor is the product of the concentration of moving charged particles, the magnitude of charge of each such particle, the magnitude of the drift velocity, and the cross-sectional area of the conductor.

We can also define a *vector* current density $\vec{J}$ that includes the direction of the drift velocity:

Vector current density: $\vec{J} = nq\vec{v}_d$ (25.4)

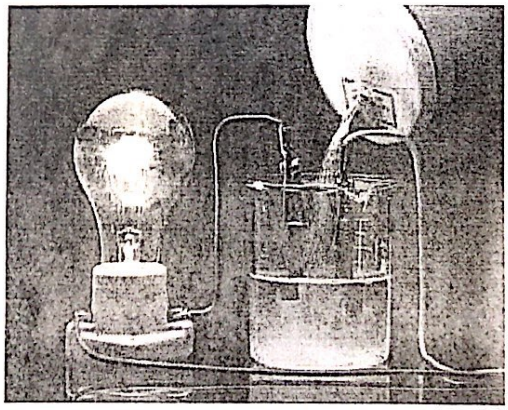
Drift velocity; Concentration of moving charged particles; Charge per particle

There are *no* absolute value signs in Eq. (25.4). If q is positive, $\vec{v}_d$ is in the same direction as $\vec{E}$; if q is negative, $\vec{v}_d$ is opposite to $\vec{E}$. In either case, $\vec{J}$ is in the same direction as $\vec{E}$. Equation (25.3) gives the *magnitude* J of the vector current density $\vec{J}$.

CAUTION **Current density vs. current** Current density $\vec{J}$ is a vector, but current I is not. The difference is that the current density $\vec{J}$ describes how charges flow at a certain point, and the vector's direction tells you about the direction of the flow at that point. By contrast, the current I describes how charges flow through an extended object such as a wire. For example, I has the same value at all points in the circuit of Fig. 25.3, but $\vec{J}$ does not: $\vec{J}$ is directed downward in the left-hand side of the loop and upward in the right-hand side. The magnitude of $\vec{J}$ can also vary around a circuit. In Fig. 25.3 $J = I/A$ is less in the battery (which has a large cross-sectional area A) than in the wires (which have a small A). ■

In general, a conductor may contain several different kinds of moving charged particles having charges $q_1, q_2, \ldots$, concentrations $n_1, n_2, \ldots$, and drift velocities with magnitudes $v_{d1}, v_{d2}, \ldots$. An example is current flow in an ionic solution (**Fig. 25.4**). In a sodium chloride solution, current can be carried by both positive sodium ions and negative chlorine ions; the total current I is found by adding up the currents due to each kind of charged particle, from Eq. (25.2). Likewise, the total vector current density $\vec{J}$ is found by using Eq. (25.4) for each kind of charged particle and adding the results.

25.4 Part of the electric circuit that includes this light bulb passes through a beaker with a solution of sodium chloride. The current in the solution is carried by both positive charges (Na^+ ions) and negative charges (Cl^- ions).



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We will see in Section 25.4 that it is possible to have a current that is *steady* (constant in time) only if the conducting material forms a closed loop, called a *complete circuit*. In such a steady situation, the total charge in every segment of the conductor is constant. Hence the rate of flow of charge *out* at one end of a segment at any instant equals the rate of flow of charge *in* at the other end of the segment, and *the current is the same at all cross sections of the circuit*. We'll use this observation when we analyze electric circuits later in this chapter.

In many simple circuits, such as flashlights or cordless electric drills, the direction of the current is always the same; this is called *direct current*. But home appliances such as toasters, refrigerators, and televisions use *alternating current*, in which the current continuously changes direction. In this chapter we'll consider direct current only. Alternating current has many special features worthy of detailed study, which we'll examine in Chapter 31.

EXAMPLE 25.1 CURRENT DENSITY AND DRIFT VELOCITY IN A WIRE

An 18-gauge copper wire (the size usually used for lamp cords), with a diameter of 1.02 mm, carries a constant current of 1.67 A to a 200-W lamp. The free-electron density in the wire is 8.5×10^{28} per cubic meter. Find (a) the current density and (b) the drift speed.

SOLUTION

IDENTIFY and SET UP: This problem uses the relationships among current I, current density J, and drift speed v_d. We are given I and the wire diameter d, so we use Eq. (25.3) to find J. We use Eq. (25.3) again to find v_d from J and the known electron density n.

EXECUTE: (a) The cross-sectional area is

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.02 \times 10^{-3}\ \text{m})^2}{4} = 8.17 \times 10^{-7}\ \text{m}^2$$

The magnitude of the current density is then

$$J = \frac{I}{A} = \frac{1.67\ \text{A}}{8.17 \times 10^{-7}\ \text{m}^2} = 2.04 \times 10^6\ \text{A/m}^2$$

(b) From Eq. (25.3) for the drift velocity magnitude v_d, we find

$$v_d = \frac{J}{n|q|} = \frac{2.04 \times 10^6\ \text{A/m}^2}{(8.5 \times 10^{28}\ \text{m}^{-3})|-1.60 \times 10^{-19}\ \text{C}|}$$
$$= 1.5 \times 10^{-4}\ \text{m/s} = 0.15\ \text{mm/s}$$

EVALUATE: At this speed an electron would require 6700 s (almost 2 h) to travel 1 m along this wire. The speeds of random motion of the electrons are roughly 10^6 m/s, around 10^{10} times the drift speed. Picture the electrons as bouncing around frantically, with a very slow drift!

TEST YOUR UNDERSTANDING OF SECTION 25.1 Suppose we replaced the wire in Example 25.1 with 12-gauge copper wire, which has twice the diameter of 18-gauge wire. If the current remains the same, what effect would this have on the magnitude of the drift velocity v_d? (i) None—v_d would be unchanged; (ii) v_d would be twice as great; (iii) v_d would be four times greater; (iv) v_d would be half as great; (v) v_d would be one-fourth as great. ❙

25.2 RESISTIVITY

The current density $\vec{J}$ in a conductor depends on the electric field $\vec{E}$ and on the properties of the material. In general, this dependence can be quite complex. But for some materials, especially metals, at a given temperature, $\vec{J}$ is nearly *directly proportional* to $\vec{E}$, and the ratio of the magnitudes of E and J is constant. This relationship, called Ohm's law, was discovered in 1826 by the German physicist Georg Simon Ohm (1787–1854). The word "law" should actually be in quotation marks, since Ohm's law, like the ideal-gas equation and Hooke's law, is an *idealized model* that describes the behavior of some materials quite well but is not a general description of *all* matter. In the following discussion we'll assume that Ohm's law is valid, even though there are many situations in which it is not.

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TABLE 25.1 Resistivities at Room Temperature (20°C)

	Substance	ρ ($\Omega \cdot$ m)	Substance	ρ ($\Omega \cdot$ m)
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

We define the **resistivity** ρ of a material as

$$\rho = \frac{E}{J} \qquad (25.5)$$

Resistivity of a material; E: Magnitude of electric field in material; J: Magnitude of current density caused by electric field

The greater the resistivity, the greater the field needed to cause a given current density, or the smaller the current density caused by a given field. From Eq. (25.5) the units of ρ are $(\text{V/m})/(\text{A/m}^2) = \text{V} \cdot \text{m/A}$. As we will discuss in Section 25.3, 1 V/A is called one *ohm* (1 Ω; the Greek letter Ω, omega, is alliterative with "ohm"). So the SI units for ρ are $\Omega \cdot$ m (ohm-meters). **Table 25.1** lists some representative values of resistivity. A perfect conductor would have zero resistivity, and a perfect insulator would have an infinite resistivity. Metals and alloys have the smallest resistivities and are the best conductors. The resistivities of insulators are greater than those of the metals by an enormous factor, on the order of 10^{22}.

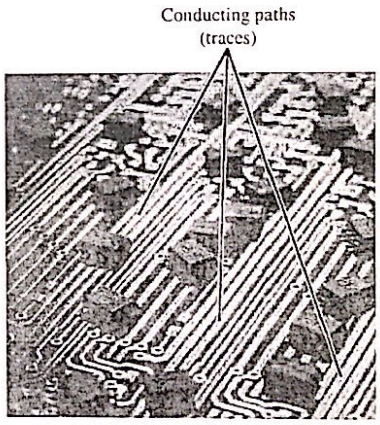
The reciprocal of resistivity is **conductivity.** Its units are $(\Omega \cdot \text{m})^{-1}$. Good conductors of electricity have larger conductivity than insulators. Conductivity is the direct electrical analog of thermal conductivity. Comparing Table 25.1 with Table 17.5 (Thermal Conductivities), we note that good electrical conductors, such as metals, are usually also good conductors of heat. Poor electrical conductors, such as ceramic and plastic materials, are also poor thermal conductors. In a metal the free electrons that carry charge in electrical conduction also provide the principal mechanism for heat conduction, so we should expect a correlation between electrical and thermal conductivity. Because of the enormous difference in conductivity between electrical conductors and insulators, it is easy to confine electric currents to well-defined paths or circuits (Fig. 25.5). The variation in *thermal* conductivity is much less, only a factor of 10^3 or so, and it is usually impossible to confine heat currents to that extent.

Semiconductors have resistivities intermediate between those of metals and those of insulators. These materials are important because of the way their resistivities are affected by temperature and by small amounts of impurities.

A material that obeys Ohm's law reasonably well is called an *ohmic* conductor or a *linear* conductor. For such materials, at a given temperature, ρ is a *constant* that does not depend on the value of E. Many materials show substantial departures from Ohm's-law behavior; they are *nonohmic*, or *nonlinear*. In these materials, J depends on E in a more complicated manner.

Analogies with fluid flow can be a big help in developing intuition about electric current and circuits. For example, in the making of wine or maple syrup, the product is sometimes filtered to remove sediments. A pump forces the fluid

25.5 The copper "wires," or traces, on this circuit board are printed directly onto the surface of the dark-colored insulating board. Even though the traces are very close to each other (only about a millimeter apart), the board has such a high resistivity (and low conductivity) that essentially no current can flow between the traces.



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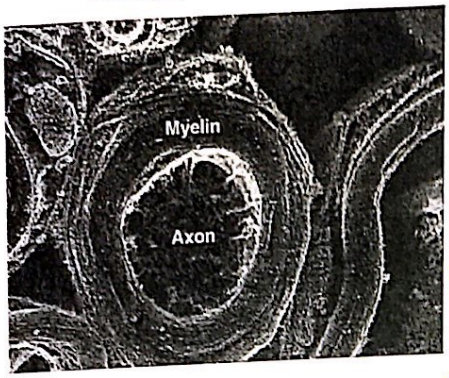
BIO Application Resistivity and Nerve Conduction This false-color image from an electron microscope shows a cross section through a nerve fiber about 1 μm (10^{-6} m) in diameter. A layer of an insulating fatty substance called myelin is wrapped around the conductive material of the axon. The resistivity of myelin is much greater than that of the axon, so an electric signal traveling along the nerve fiber remains confined to the axon. This makes it possible for a signal to travel much more rapidly than if the myelin were absent.



through the filter under pressure; if the flow rate (analogous to J) is proportional to the pressure difference between the upstream and downstream sides (analogous to E), the behavior is analogous to Ohm's law.

Resistivity and Temperature

The resistivity of a *metallic* conductor nearly always increases with increasing temperature, as shown in **Fig. 25.6a**. As temperature increases, the ions of the conductor vibrate with greater amplitude, making it more likely that a moving electron will collide with an ion as in Fig. 25.1; this impedes the drift of electrons through the conductor and hence reduces the current. Over a small temperature range (up to 100 C° or so), the resistivity of a metal can be represented approximately by the equation

Temperature dependence of resistivity:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \qquad (25.6)$$

($\rho(T)$: resistivity at temperature T; α: temperature coefficient of resistivity; ρ_0: resistivity at reference temperature T_0)

The reference temperature T_0 is often taken as 0°C or 20°C; the temperature T may be higher or lower than T_0. The factor α is called the **temperature coefficient of resistivity.** Some representative values are given in **Table 25.2.** The resistivity of the alloy manganin is practically independent of temperature.

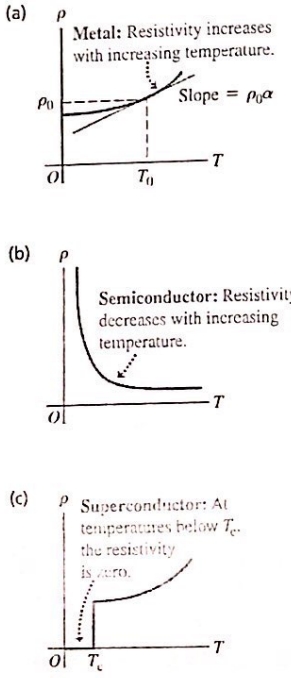
The resistivity of graphite (a nonmetal) *decreases* with increasing temperature, since at higher temperatures, more electrons "shake loose" from the atoms and become mobile; hence the temperature coefficient of resistivity of graphite is negative. The same behavior occurs for semiconductors (Fig. 25.6b). Measuring the resistivity of a small semiconductor crystal is therefore a sensitive measure of temperature; this is the principle of a type of thermometer called a *thermistor.*

Some materials, including several metallic alloys and oxides, show a phenomenon called *superconductivity.* As the temperature decreases, the resistivity at first decreases smoothly, like that of any metal. But then at a certain critical temperature T_c a phase transition occurs and the resistivity suddenly drops to zero, as shown in Fig. 25.6c. Once a current has been established in a superconducting ring, it continues indefinitely without the presence of any driving field.

Superconductivity was discovered in 1911 by the Dutch physicist Heike Kamerlingh Onnes (1853–1926). He discovered that at very low temperatures, below 4.2 K, the resistivity of mercury suddenly dropped to zero. For the next 75 years, the highest T_c attained was about 20 K. This meant that superconductivity occurred only when the material was cooled by using expensive liquid helium, with a boiling-point temperature of 4.2 K, or explosive liquid hydrogen, with a boiling point of 20.3 K. But in 1986 Karl Müller and Johannes Bednorz discovered an oxide of barium, lanthanum, and copper with a T_c of nearly 40 K, and the race was on to develop "high-temperature" superconducting materials.

By 1987 a complex oxide of yttrium, copper, and barium had been found that has a value of T_c well above the 77 K boiling temperature of liquid nitrogen, a refrigerant that is both inexpensive and safe. The current (2014) record for T_c

25.6 Variation of resistivity ρ with absolute temperature T for (a) a normal metal, (b) a semiconductor, and (c) a superconductor. In (a) the linear approximation to ρ as a function of T is shown as a green line; the approximation agrees exactly at $T = T_0$, where $\rho = \rho_0$.



Temperature Coefficients of Resistivity (Approximate Values Near Room Temperature) — TABLE 25.2

Material	α [(°C)$^{-1}$]	Material	α [(°C)$^{-1}$]
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

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at atmospheric pressure is 138 K, and materials that are superconductors at room temperature may become a reality. The implications of these discoveries for power-distribution systems, computer design, and transportation are enormous. Meanwhile, superconducting electromagnets cooled by liquid helium are used in particle accelerators and some experimental magnetic-levitation railroads. Superconductors have other exotic properties that require an understanding of magnetism to explore; we will discuss these further in Chapter 29.

TEST YOUR UNDERSTANDING OF SECTION 25.2 You maintain a constant electric field inside a piece of semiconductor while lowering the semiconductor's temperature. What happens to the current density in the semiconductor? (i) It increases; (ii) it decreases; (iii) it remains the same. I

25.3 RESISTANCE

For a conductor with resistivity ρ, the current density $\vec{J}$ at a point where the electric field is $\vec{E}$ is given by Eq. (25.5), which we can write as

$$\vec{E} = \rho\vec{J} \tag{25.7}$$

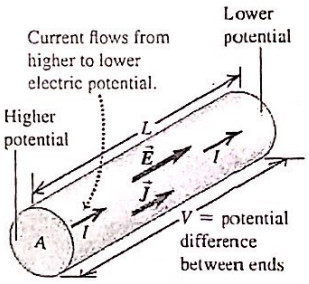
MP

PhET: Resistance in a Wire

When Ohm's law is obeyed, ρ is constant and independent of the magnitude of the electric field, so $\vec{E}$ is directly proportional to $\vec{J}$. Often, however, we are more interested in the total current I in a conductor than in $\vec{J}$ and more interested in the potential difference V between the ends of the conductor than in $\vec{E}$. This is so largely because I and V are much easier to measure than are $\vec{J}$ and $\vec{E}$.

Suppose our conductor is a wire with uniform cross-sectional area A and length L, as shown in **Fig. 25.7**. Let V be the potential difference between the higher-potential and lower-potential ends of the conductor, so that V is positive. (Another name for V is the *voltage across* the conductor.) The *direction* of the current is always from the higher-potential end to the lower-potential end. That's because current in a conductor flows in the direction of $\vec{E}$, no matter what the sign of the moving charges (Fig. 25.2), and because $\vec{E}$ points in the direction of *decreasing* electric potential (see Section 23.2). As the current flows through the potential difference, electric potential energy is lost; this energy is transferred to the ions of the conducting material during collisions.

25.7 A conductor with uniform cross section. The current density is uniform over any cross section, and the electric field is constant along the length.



We can also relate the *value* of the current I to the potential difference between the ends of the conductor. If the magnitudes of the current density $\vec{J}$ and the electric field $\vec{E}$ are uniform throughout the conductor, the total current I is $I = JA$, and the potential difference V between the ends is $V = EL$. We solve these equations for J and E, respectively, and substitute the results into Eq. (25.7):

$$\frac{V}{L} = \frac{\rho I}{A} \quad \text{or} \quad V = \frac{\rho L}{A} I \tag{25.8}$$

This shows that when ρ is constant, the total current I is proportional to the potential difference V.

The ratio of V to I for a particular conductor is called its **resistance** R:

$$R = \frac{V}{I} \tag{25.9}$$

Comparing this definition of R to Eq. (25.8), we see that

$$\text{Resistance of a conductor: } R = \frac{\rho L}{A} \tag{25.10}$$

(ρ: Resistivity of conductor material; L: Length of conductor; A: Cross-sectional area of conductor)

If ρ is constant, as is the case for ohmic materials, then so is R.

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The following equation is often called Ohm's law:

Relationship among voltage, current, and resistance:

$$V = IR \quad (25.11)$$

(V: voltage between ends of conductor; R: resistance of conductor; I: current in conductor)

However, it's important to understand that the real content of Ohm's law is the direct proportionality (for some materials) of V to I or of J to E. Equation (25.9) or (25.11) *defines* resistance R for *any* conductor, but only when R is constant can we correctly call this relationship Ohm's law.

25.8 A long fire hose offers substantial resistance to water flow. To make water pass through the hose rapidly, the upstream end of the hose must be at much higher pressure than the end where the water emerges. In an analogous way, there must be a large potential difference between the ends of a long wire in order to cause a substantial electric current through the wire.

Interpreting Resistance

Equation (25.10) shows that the resistance of a wire or other conductor of uniform cross section is directly proportional to its length and inversely proportional to its cross-sectional area. It is also proportional to the resistivity of the material of which the conductor is made.

The flowing-fluid analogy is again useful. In analogy to Eq. (25.10), a narrow water hose offers more resistance to flow than a fat one, and a long hose has more resistance than a short one (**Fig. 25.8**). We can increase the resistance to flow by stuffing the hose with cotton or sand; this corresponds to increasing the resistivity. The flow rate is approximately proportional to the pressure difference between the ends. Flow rate is analogous to current, and pressure difference is analogous to potential difference (voltage). Let's not stretch this analogy too far, though; the water flow rate in a pipe is usually *not* proportional to its cross-sectional area (see Section 14.6).

The SI unit of resistance is the **ohm**, equal to one volt per ampere ($1\ \Omega = 1\ \text{V/A}$). The *kilohm* ($1\ \text{k}\Omega = 10^3\ \Omega$) and the *megohm* ($1\ \text{M}\Omega = 10^6\ \Omega$) are also in common use. A 100-m length of 12-gauge copper wire, the size usually used in household wiring, has a resistance at room temperature of about $0.5\ \Omega$. A 100-W, 120-V incandescent light bulb has a resistance (at operating temperature) of $140\ \Omega$. If the same current I flows in both the copper wire and the light bulb, the potential difference $V = IR$ is much greater across the light bulb, and much more potential energy is lost per charge in the light bulb. This lost energy is converted by the light bulb filament into light and heat. You don't want your household wiring to glow white-hot, so its resistance is kept low by using wire of low resistivity and large cross-sectional area.

Because the resistivity of a material varies with temperature, the resistance of a specific conductor also varies with temperature. For temperature ranges that are not too great, this variation is approximately linear, analogous to Eq. (25.6):

$$R(T) = R_0[1 + \alpha(T - T_0)] \quad (25.12)$$

In this equation, $R(T)$ is the resistance at temperature T and R_0 is the resistance at temperature T_0, often taken to be 0°C or 20°C. The *temperature coefficient of resistance* α is the same constant that appears in Eq. (25.6) if the dimensions L and A in Eq. (25.10) do not change appreciably with temperature; this is indeed the case for most conducting materials. Within the limits of validity of Eq. (25.12), the *change* in resistance resulting from a temperature change $T - T_0$ is given by $R_0\alpha(T - T_0)$.

A circuit device made to have a specific value of resistance between its ends is called a **resistor**. Resistors in the range 0.01 to $10^7\ \Omega$ can be bought off the shelf. Individual resistors used in electronic circuitry are often cylindrical, a few millimeters in diameter and length, with wires coming out of the ends. The resistance may be marked with a standard code that uses three or four color bands near one end (**Fig. 25.9**), according to the scheme in **Table 25.3**. The first two bands (starting with the band nearest an end) are digits, and the third is a power-of-10 multiplier. For example, green–violet–red means $57 \times 10^2\ \Omega$, or $5.7\ \text{k}\Omega$. The fourth band, if present, indicates the accuracy (tolerance) of the value;

25.9 This resistor has a resistance of $5.7\ \text{k}\Omega$ with an accuracy (tolerance) of $\pm 10\%$.

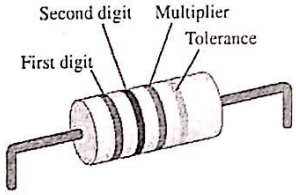

TABLE 25.3 Color Codes for Resistors

Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

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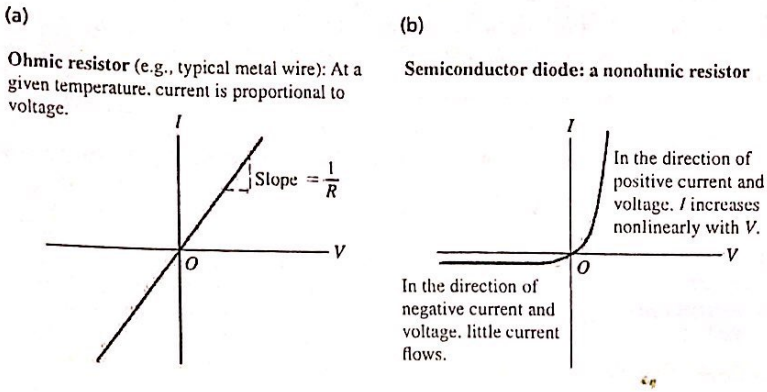

25.10 Current–voltage relationships for two devices. Only for a resistor that obeys Ohm's law as in (a) is current I proportional to voltage V.

no band means ±20%, a silver band ±10%, and a gold band ±5%. Another important characteristic of a resistor is the maximum *power* it can dissipate without damage. We'll return to this point in Section 25.5.

For a resistor that obeys Ohm's law, a graph of current as a function of potential difference (voltage) is a straight line (**Fig. 25.10a**). The slope of the line is $1/R$. If the sign of the potential difference changes, so does the sign of the current produced; in Fig. 25.7 this corresponds to interchanging the higher- and lower-potential ends of the conductor, so the electric field, current density, and current all reverse direction.

In devices that do not obey Ohm's law, the relationship of voltage to current may not be a direct proportion, and it may be different for the two directions of current. Figure 25.10b shows the behavior of a semiconductor *diode*, a device used to convert alternating current to direct current and to perform a wide variety of logic functions in computer circuitry. For positive potentials V of the anode (one of two terminals of the diode) with respect to the cathode (the other terminal), I increases exponentially with increasing V; for negative potentials the current is extremely small. Thus a positive V causes a current to flow in the positive direction, but a potential difference of the other sign causes little or no current. Hence a diode acts like a one-way valve in a circuit.

EXAMPLE 25.2 ELECTRIC FIELD, POTENTIAL DIFFERENCE, AND RESISTANCE IN A WIRE

The 18-gauge copper wire of Example 25.1 has a cross-sectional area of $8.20 \times 10^{-7}\ \text{m}^2$. It carries a current of 1.67 A. Find (a) the electric-field magnitude in the wire; (b) the potential difference between two points in the wire 50.0 m apart; (c) the resistance of a 50.0-m length of this wire.

SOLUTION

IDENTIFY and SET UP: We are given the cross-sectional area A and current I. Our target variables are the electric-field magnitude E, potential difference V, and resistance R. The current density is $J = I/A$. We find E from Eq. (25.5), $E = \rho J$ (Table 25.1 gives the resistivity ρ for copper). The potential difference is then the product of E and the length of the wire. We can use either Eq. (25.10) or Eq. (25.11) to find R.

EXECUTE: (a) From Table 25.1, $\rho = 1.72 \times 10^{-8}\ \Omega \cdot \text{m}$. Hence, from Eq. (25.5),

$$E = \rho J = \frac{\rho I}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(1.67\ \text{A})}{8.20 \times 10^{-7}\ \text{m}^2} = 0.0350\ \text{V/m}$$

(b) The potential difference is

$$V = EL = (0.0350\ \text{V/m})(50.0\ \text{m}) = 1.75\ \text{V}$$

(c) From Eq. (25.10) the resistance of 50.0 m of this wire is

$$R = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(50.0\ \text{m})}{8.20 \times 10^{-7}\ \text{m}^2} = 1.05\ \Omega$$

Alternatively, we can find R from Eq. (25.11):

$$R = \frac{V}{I} = \frac{1.75\ \text{V}}{1.67\ \text{A}} = 1.05\ \Omega$$

EVALUATE: We emphasize that the resistance of the wire is *defined* to be the ratio of voltage to current. If the wire is made of nonohmic material, then R is different for different values of V but is always given by $R = V/I$. Resistance is also always given by $R = \rho L/A$; if the material is nonohmic, ρ is not constant but depends on E (or, equivalently, on $V = EL$).

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EXAMPLE 25.3 TEMPERATURE DEPENDENCE OF RESISTANCE

SOLUTION

Suppose the resistance of a copper wire is 1.05 Ω at 20°C. Find the resistance at 0°C and 100°C.

SOLUTION

IDENTIFY and SET UP: We are given the resistance $R_0 = 1.05\ \Omega$ at a reference temperature $T_0 = 20°C$. We use Eq. (25.12) to find the resistances at $T = 0°C$ and $T = 100°C$ (our target variables), taking the temperature coefficient of resistivity from Table 25.2.

EXECUTE: From Table 25.2, $\alpha = 0.00393\ (C°)^{-1}$ for copper. Then from Eq. (25.12),

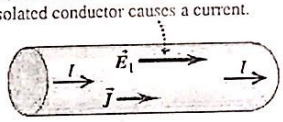
$$\begin{aligned} R &= R_0[1 + \alpha(T - T_0)] \\ &= (1.05\ \Omega)\{1 + [0.00393\ (C°)^{-1}][0°C - 20°C]\} \\ &= 0.97\ \Omega \text{ at } T = 0°C \\ R &= (1.05\ \Omega)\{1 + [0.00393\ (C°)^{-1}][100°C - 20°C]\} \\ &= 1.38\ \Omega \text{ at } T = 100°C \end{aligned}$$

EVALUATE: The resistance at 100°C is greater than that at 0°C by a factor of $(1.38\ \Omega)/(0.97\ \Omega) = 1.42$: Raising the temperature of copper wire from 0°C to 100°C increases its resistance by 42%. From Eq. (25.11), $V = IR$, this means that 42% more voltage is required to produce the same current at 100°C than at 0°C. Designers of electric circuits that must operate over a wide temperature range must take this substantial effect into account.

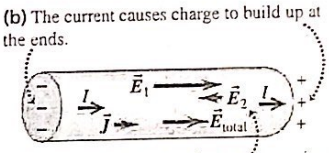
TEST YOUR UNDERSTANDING OF SECTION 25.3 Suppose you increase the voltage across the copper wire in Examples 25.2 and 25.3. The increased voltage causes more current to flow, which makes the temperature of the wire increase. (The same thing happens to the coils of an electric oven or a toaster when a voltage is applied to them. We'll explore this issue in more depth in Section 25.5.) If you double the voltage across the wire, the current in the wire increases. By what factor does it increase? (i) 2; (ii) greater than 2; (iii) less than 2. **I**

25.11 If an electric field is produced inside a conductor that is *not* part of a complete circuit, current flows for only a very short time.

(a) An electric field $\vec{E}_1$ produced inside an isolated conductor causes a current.

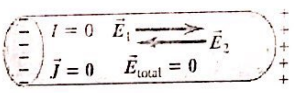


(b) The current causes charge to build up at the ends.



The charge buildup produces an opposing field $\vec{E}_2$, thus reducing the current.

(c) After a very short time $\vec{E}_2$ has the same magnitude as $\vec{E}_1$; then the total field is $\vec{E}_{\text{total}} = 0$ and the current stops completely.



25.4 ELECTROMOTIVE FORCE AND CIRCUITS

For a conductor to have a steady current, it must be part of a path that forms a closed loop or **complete circuit**. Here's why. If you establish an electric field $\vec{E}_1$ inside an isolated conductor with resistivity ρ that is *not* part of a complete circuit, a current begins to flow with current density $\vec{J} = \vec{E}_1/\rho$ (**Fig. 25.11a**). As a result a net positive charge quickly accumulates at one end of the conductor and a net negative charge accumulates at the other end (Fig. 25.11b). These charges themselves produce an electric field $\vec{E}_2$ in the direction opposite to $\vec{E}_1$, causing the total electric field and hence the current to decrease. Within a very small fraction of a second, enough charge builds up on the conductor ends that the total electric field $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0$ inside the conductor. Then $\vec{J} = 0$ as well, and the current stops altogether (Fig. 25.11c). So there can be no steady motion of charge in such an *incomplete* circuit.

To see how to maintain a steady current in a *complete* circuit, we recall a basic fact about electric potential energy: If a charge q goes around a complete circuit and returns to its starting point, the potential energy must be the same at the end of the round trip as at the beginning. As described in Section 25.3, there is always a *decrease* in potential energy when charges move through an ordinary conducting material with resistance. So there must be some part of the circuit in which the potential energy *increases*.

The problem is analogous to an ornamental water fountain that recycles its water. The water pours out of openings at the top, cascades down over the terraces and spouts (moving in the direction of decreasing gravitational potential energy),

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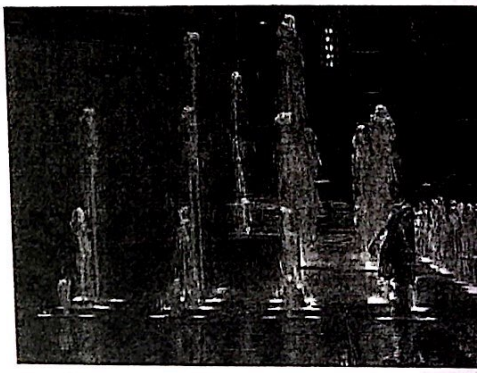
and collects in a basin in the bottom. A pump then lifts it back to the top (increasing the potential energy) for another trip. Without the pump, the water would just fall to the bottom and stay there.

Electromotive Force

In an electric circuit there must be a device somewhere in the loop that acts like the water pump in a water fountain (**Fig. 25.12**). In this device a charge travels "uphill," from lower to higher potential energy, even though the electrostatic force is trying to push it from higher to lower potential energy. The direction of current in such a device is from lower to higher potential, just the opposite of what happens in an ordinary conductor.

25.12 Just as a water fountain requires a pump, an electric circuit requires a source of electromotive force to sustain a steady current.

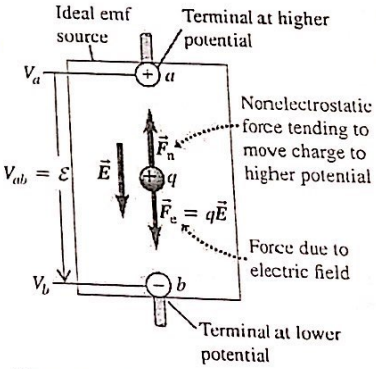


The influence that makes current flow from lower to higher potential is called **electromotive force** (abbreviated **emf** and pronounced "ee-em-eff"), and a circuit device that provides emf is called a **source of emf.** Note that "electromotive force" is a poor term because emf is *not* a force but an energy-per-unit-charge quantity, like potential. The SI unit of emf is the same as that for potential, the volt (1 V = 1 J/C). A typical flashlight battery has an emf of 1.5 V; this means that the battery does 1.5 J of work on every coulomb of charge that passes through it. We'll use the symbol $\mathcal{E}$ (a script capital E) for emf.

Every complete circuit with a steady current must include a source of emf. Batteries, electric generators, solar cells, thermocouples, and fuel cells are all examples of sources of emf. All such devices convert energy of some form (mechanical, chemical, thermal, and so on) into electric potential energy and transfer it into the circuit to which the device is connected. An *ideal* source of emf maintains a constant potential difference between its terminals, independent of the current through it. We define electromotive force quantitatively as the magnitude of this potential difference. As we will see, such an ideal source is a mythical beast, like the frictionless plane and the massless rope. We will discuss later how real-life sources of emf differ in their behavior from this idealized model.

Figure 25.13 is a schematic diagram of an ideal source of emf that maintains a potential difference between conductors a and b, called the *terminals* of the device. Terminal a, marked +, is maintained at *higher* potential than terminal b, marked −. Associated with this potential difference is an electric field $\vec{E}$ in the region around the terminals, both inside and outside the source. The electric field inside the device is directed from a to b, as shown. A charge q within the source experiences an electric force $\vec{F}_e = q\vec{E}$. But the source also provides an additional influence, which we represent as a nonelectrostatic force $\vec{F}_n$. This force, operating inside the device, pushes charge from b to a in an "uphill" direction against the electric force $\vec{F}_e$. Thus $\vec{F}_n$ maintains the potential difference between the terminals. If $\vec{F}_n$ were not present, charge would flow between the terminals until the potential difference was zero. The origin of the additional influence $\vec{F}_n$ depends on the kind of source. In a generator it results from magnetic-field forces on moving charges. In a battery or fuel cell it is associated with diffusion processes and varying electrolyte concentrations resulting from chemical reactions. In an electrostatic machine such as a Van de Graaff generator (see Fig. 22.26), an actual mechanical force is applied by a moving belt or wheel.

25.13 Schematic diagram of a source of emf in an "open-circuit" situation. The electric-field force $\vec{F}_e = q\vec{E}$ and the nonelectrostatic force $\vec{F}_n$ are shown for a positive charge q.



When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

If a positive charge q is moved from b to a inside the source, the nonelectrostatic force $\vec{F}_n$ does a positive amount of work $W_n = q\mathcal{E}$ on the charge. This displacement is *opposite* to the electrostatic force $\vec{F}_e$, so the potential energy associated with the charge *increases* by an amount equal to qV_{ab}, where $V_{ab} = V_a - V_b$ is the (positive) potential of point a with respect to point b. For the ideal source of emf that we've described, $\vec{F}_e$ and $\vec{F}_n$ are equal in magnitude but opposite in direction, so the total work done on the charge q is zero; there is an increase in potential energy but *no* change in the kinetic energy of the charge.

MP

PhET: Battery Voltage
PhET: Signal Circuit

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It's like lifting a book from the floor to a high shelf at constant speed. The increase in potential energy is just equal to the nonelectrostatic work W_n, so $q\mathcal{E} = qV_{ab}$, or

$$V_{ab} = \mathcal{E} \quad \text{(ideal source of emf)} \tag{25.13}$$

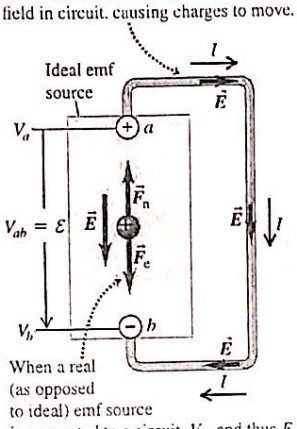
Now let's make a complete circuit by connecting a wire with resistance R to the terminals of a source (**Fig. 25.14**). The potential difference between terminals a and b sets up an electric field within the wire; this causes current to flow around the loop from a toward b, from higher to lower potential. Where the wire bends, equal amounts of positive and negative charge persist on the "inside" and "outside" of the bend. These charges exert the forces that cause the current to follow the bends in the wire.

From Eq. (25.11) the potential difference between the ends of the wire in Fig. 25.14 is given by $V_{ab} = IR$. Combining with Eq. (25.13), we have

$$\mathcal{E} = V_{ab} = IR \quad \text{(ideal source of emf)} \tag{25.14}$$

That is, when a positive charge q flows around the circuit, the potential *rise* $\mathcal{E}$ as it passes through the ideal source is numerically equal to the potential *drop* $V_{ab} = IR$ as it passes through the remainder of the circuit. Once $\mathcal{E}$ and R are known, this relationship determines the current in the circuit.

25.14 Schematic diagram of an ideal source of emf in a complete circuit. The electric-field force $\vec{F}_e = q\vec{E}$ and the nonelectrostatic force $\vec{F}_n$ are shown for a positive charge q. The current is in the direction from a to b in the external circuit and from b to a within the source.



CAUTION Current is not "used up" in a circuit It's a common misconception that in a closed circuit, current squirts out of the positive terminal of a battery and is consumed or "used up" by the time it reaches the negative terminal. In fact the current is the *same* at every point in a simple loop circuit like that in Fig. 25.14, even if the wire thickness is not constant throughout the circuit. This happens because charge is conserved (it can be neither created nor destroyed) and because charge cannot accumulate in the circuit devices we have described. It's like the flow of water in an ornamental fountain; water flows out of the top at the same rate at which it reaches the bottom, no matter what the dimensions of the fountain. None of the water is "used up" along the way! ?

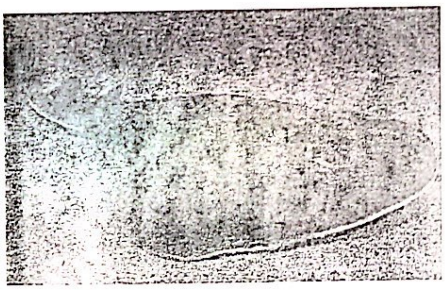
Internal Resistance

Real sources of emf in a circuit don't behave in exactly the way we have described; the potential difference across a real source in a circuit is *not* equal to the emf as in Eq. (25.14). The reason is that charge moving through the material of any real source encounters *resistance*. We call this the **internal resistance** of the source, denoted by r. If this resistance behaves according to Ohm's law, r is constant and independent of the current I. As the current moves through r, it experiences an associated drop in potential equal to Ir. Thus, when a current is flowing through a source from the negative terminal b to the positive terminal a, the potential difference V_{ab} between the terminals is

$$V_{ab} = \mathcal{E} - Ir \tag{25.15}$$

Terminal voltage of source with internal resistance — emf of source — Current through source — internal resistance of source

BIO Application Danger: Electric Ray! Electric rays deliver electric shocks to stun their prey and to discourage predators. (In ancient Rome, physicians practiced a primitive form of electroconvulsive therapy by placing electric rays on their patients to cure headaches and gout.) The shocks are produced by specialized flattened cells called electroplaques. Such a cell moves ions across membranes to produce an emf of about 0.05 V. Thousands of electroplaques are stacked on top of each other, so their emfs add to a total of as much as 200 V. These stacks make up more than half of an electric ray's body mass. A ray can use these to deliver an impressive current of up to 30 A for a few milliseconds.



The potential V_{ab}, called the terminal voltage, is less than the emf $\mathcal{E}$ because of the term Ir representing the potential drop across the internal resistance r. Hence the increase in potential energy qV_{ab} as a charge q moves from b to a within the source is less than the work $q\mathcal{E}$ done by the nonelectrostatic force $\vec{F}_n$, since some potential energy is lost in traversing the internal resistance.

A 1.5-V battery has an emf of 1.5 V, but the terminal voltage V_{ab} of the battery is equal to 1.5 V only if no current is flowing through it so that $I = 0$ in Eq. (25.15). If the battery is part of a complete circuit through which current

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is flowing, the terminal voltage will be less than 1.5 V. *For a real source of emf, the terminal voltage equals the emf only if no current is flowing through the source* (**Fig. 25.15**). Thus we can describe the behavior of a source in terms of two properties: an emf $\mathcal{E}$, which supplies a constant potential difference independent of current, in series with an internal resistance r.

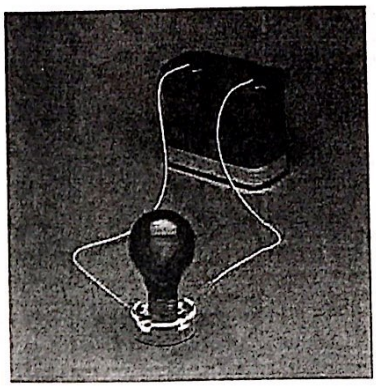
The current in the external circuit connected to the source terminals a and b is still determined by $V_{ab} = IR$. Combining this with Eq. (25.15), we find

$$\mathcal{E} - Ir = IR \quad \text{or} \quad I = \frac{\mathcal{E}}{R + r} \qquad \text{(current, source with internal resistance)} \qquad (25.16)$$

That is, the current equals the source emf divided by the *total* circuit resistance $(R + r)$.

CAUTION A battery is not a "current source" You might have thought that a battery or other source of emf always produces the same current, no matter what circuit it's used in. Equation (25.16) shows that this isn't so! The greater the resistance R of the external circuit, the less current the source will produce. ❙

25.15 The emf of this battery—that is, the terminal voltage when it's not connected to anything—is 12 V. But because the battery has internal resistance, the terminal voltage of the battery is less than 12 V when it is supplying current to a light bulb.

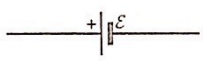
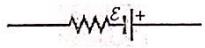
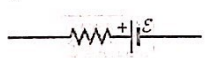
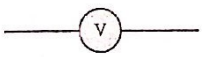
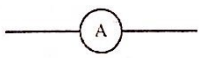


Symbols for Circuit Diagrams

An important part of analyzing any electric circuit is drawing a schematic *circuit diagram*. **Table 25.4** shows the usual symbols used in circuit diagrams. We will use these symbols extensively in this chapter and the next. We usually assume that the wires that connect the various elements of the circuit have negligible resistance; from Eq. (25.11), $V = IR$, the potential difference between the ends of such a wire is zero.

Table 25.4 includes two *meters* that are used to measure the properties of circuits. Idealized meters do not disturb the circuit in which they are connected. A **voltmeter**, introduced in Section 23.2, measures the potential difference between its terminals; an idealized voltmeter has infinitely large resistance and measures potential difference without having any current diverted through it. An ammeter measures the current passing through it; an idealized **ammeter** has zero resistance and has no potential difference between its terminals. The following examples illustrate how to analyze circuits that include meters.

TABLE 25.4 Symbols for Circuit Diagrams

	Conductor with negligible resistance
	Resistor
	Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)
 or 	Source of emf with internal resistance r (r can be placed on either side)
	Voltmeter (measures potential difference between its terminals)
	Ammeter (measures current through it)

DATA *SPEAKS*

Circuits, emf, and Current

When students were given a problem involving a source of emf, more than 25% gave an incorrect response. Common errors:

- Forgetting that the internal resistance r affects the potential difference V_{ab} between the terminals of the source of emf $\mathcal{E}$. If the current I inside the source is from the negative terminal b to the positive terminal a, then $V_{ab} < \mathcal{E}$ by an amount Ir; if the current flows the other way, $V_{ab} > \mathcal{E}$ by an amount Ir.
- Forgetting that the internal resistance is an intrinsic part of a source of emf. Although we draw the emf and internal resistance as adjacent parts of the circuit, both are parts of the source and cannot be separated.

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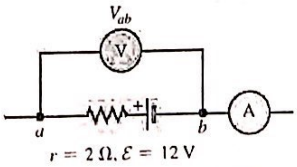
CONCEPTUAL EXAMPLE 25.4 A SOURCE IN AN OPEN CIRCUIT

Figure 25.16 shows a source (a battery) with emf $\mathcal{E} = 12$ V and internal resistance $r = 2\ \Omega$. (For comparison, the internal resistance of a commercial 12-V lead storage battery is only a few thousandths of an ohm.) The wires to the left of a and to the right of the ammeter A are not connected to anything. Determine the respective readings V_{ab} and I of the idealized voltmeter V and the idealized ammeter A.

25.16 A source of emf in an open circuit.



SOLUTION

There is *zero* current because there is no complete circuit. (Our idealized voltmeter has an infinitely large resistance, so no current flows through it.) Hence the ammeter reads $I = 0$. Because there is no current through the battery, there is no potential difference across its internal resistance. From Eq. (25.15) with $I = 0$, the potential difference V_{ab} across the battery terminals is equal to the emf. So the voltmeter reads $V_{ab} = \mathcal{E} = 12$ V. The terminal voltage of a real, nonideal source equals the emf *only* if there is no current flowing through the source, as in this example.

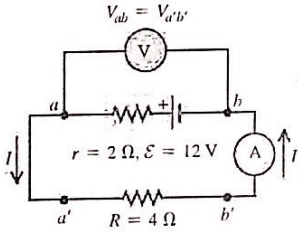
EXAMPLE 25.5 A SOURCE IN A COMPLETE CIRCUIT

We add a 4-Ω resistor to the battery in Conceptual Example 25.4, forming a complete circuit (**Fig. 25.17**). What are the voltmeter and ammeter readings V_{ab} and I now?

SOLUTION

IDENTIFY and SET UP: Our target variables are the current I through the circuit $aa'b'b$ and the potential difference V_{ab}. We first find I from Eq. (25.16). To find V_{ab}, we can use either Eq. (25.11) or Eq. (25.15).

25.17 A source of emf in a complete circuit.



EXECUTE: The ideal ammeter has zero resistance, so the total resistance external to the source is $R = 4\ \Omega$. From Eq. (25.16), the current through the circuit $aa'b'b$ is then

$$I = \frac{\mathcal{E}}{R + r} = \frac{12\text{ V}}{4\ \Omega + 2\ \Omega} = 2\text{ A}$$

Our idealized conducting wires and the idealized ammeter have zero resistance, so there is no potential difference between points a and a' or between points b and b'; that is, $V_{ab} = V_{a'b'}$. We find V_{ab} by considering a and b as the terminals of the resistor: From Ohm's law, Eq. (25.11), we then have

$$V_{a'b'} = IR = (2\text{ A})(4\ \Omega) = 8\text{ V}$$

Alternatively, we can consider a and b as the terminals of the source. Then, from Eq. (25.15),

$$V_{ab} = \mathcal{E} - Ir = 12\text{ V} - (2\text{ A})(2\ \Omega) = 8\text{ V}$$

Either way, we see that the voltmeter reading is 8 V.

EVALUATE: With this current flowing through the source, the terminal voltage V_{ab} is less than the emf $\mathcal{E}$. The smaller the internal resistance r, the less the difference between V_{ab} and $\mathcal{E}$.

CONCEPTUAL EXAMPLE 25.6 USING VOLTMETERS AND AMMETERS

We move the voltmeter and ammeter in Example 25.5 to different positions in the circuit. What are the readings of the ideal voltmeter and ammeter in the situations shown in (a) Fig. 25.18a and (b) Fig. 25.18b?

SOLUTION

(a) The voltmeter now measures the potential difference between points a' and b'. As in Example 25.5, $V_{ab} = V_{a'b'}$, so the voltmeter reads the same as in Example 25.5: $V_{a'b'} = 8$ V.

CAUTION Current in a simple loop As charges move through a resistor, there is a decrease in electric potential energy, but there is *no* change in the current. *The current in a simple loop is the same at every point;* it is not "used up" as it moves through a resistor. Hence the ammeter in Fig. 25.17 ("downstream" of the 4-Ω resistor) and the ammeter in Fig. 25.18b ("upstream" of the resistor) both read $I = 2$ A. ◀

(b) There is no current through the ideal voltmeter because it has infinitely large resistance. Since the voltmeter is now part of

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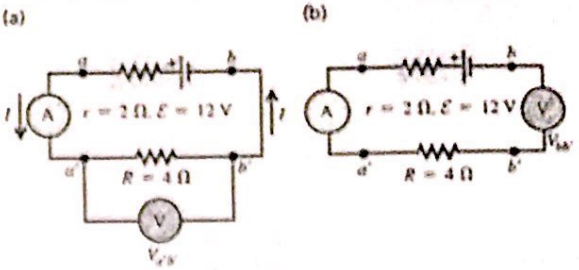
the circuit, there is no current at all in the circuit, and the ammeter reads $I = 0$.

The voltmeter measures the potential difference $V_{bb'}$ between points b and b'. Since $I = 0$, the potential difference across the resistor is $V_{a'b'} = IR = 0$, and the potential difference between the ends a and a' of the idealized ammeter is also zero. So $V_{bb'}$ is equal to V_{ab}, the terminal voltage of the source. As in Conceptual Example 25.4, there is no current, so the terminal voltage equals the emf, and the voltmeter reading is $V_{ab} = \mathcal{E} = 12$ V.

This example shows that ammeters and voltmeters are circuit elements, too. Moving the voltmeter from the position in Fig. 25.18a to that in Fig. 25.18b makes large changes in the current and potential differences in the circuit. If you want to measure the potential difference between two points in a circuit without disturbing the circuit, use a voltmeter as in Fig. 25.17 or 25.18a, *not* as in Fig. 25.18b.

25.18 Different placements of a voltmeter and an ammeter in a complete circuit.



EXAMPLE 25.7 A SOURCE WITH A SHORT CIRCUIT

SOLUTION

In the circuit of Example 25.5 we replace the 4-Ω resistor with a zero-resistance conductor. What are the meter readings now?

SOLUTION

IDENTIFY and SET UP: **Figure 25.19** shows the new circuit. Our target variables are again I and V_{ab}. There is now a zero-resistance path between points a and b, through the lower loop, so the potential difference between these points must be zero.

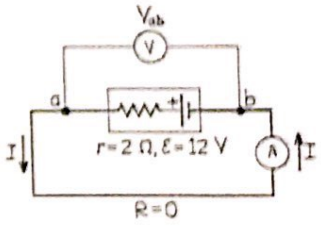
EXECUTE: We must have $V_{ab} = IR = I(0) = 0$, no matter what the current. We can therefore find the current I from Eq. (25.15):

$$V_{ab} = \mathcal{E} - Ir = 0$$

$$I = \frac{\mathcal{E}}{r} = \frac{12\text{ V}}{2\ \Omega} = 6\text{ A}$$

EVALUATE: The current in this circuit has a different value than in Example 25.5, even though the same battery is used; the current depends on both the internal resistance r and the resistance of the external circuit.

25.19 Our sketch for this problem.



The situation here is called a *short circuit*. The external-circuit resistance is zero, because terminals of the battery are connected directly to each other. The short-circuit current is equal to the emf $\mathcal{E}$ divided by the internal resistance r. *Warning:* Short circuits can be dangerous! An automobile battery or a household power line has very small internal resistance (much less than in these examples), and the short-circuit current can be great enough to melt a small wire or cause a storage battery to explode.

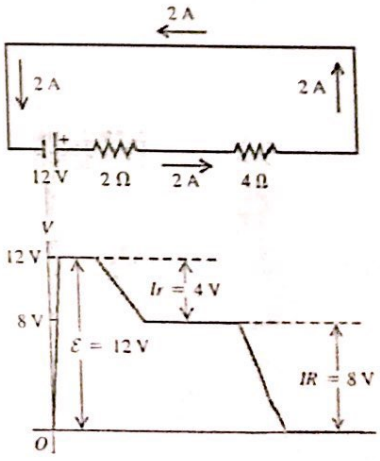
Potential Changes around a Circuit

The net change in potential energy for a charge q making a round trip around a complete circuit must be zero. Hence the net change in *potential* around the circuit must also be zero, in other words, the algebraic sum of the potential differences and emfs around the loop is zero. We can see this by rewriting Eq. (25.16) in the form

$$\mathcal{E} - Ir - IR = 0$$

A potential gain of $\mathcal{E}$ is associated with the emf, and potential drops of Ir and IR are associated with the internal resistance of the source and the external circuit, respectively. **Figure 25.20** shows how the potential varies as we go around the complete circuit of Fig. 25.17. The horizontal axis doesn't necessarily represent actual distances, but rather various points in the loop. If we take the potential to be zero at the negative terminal of the battery, then we have a rise $\mathcal{E}$ and a drop Ir in the battery and an additional drop IR in the external resistor, and as we finish our trip around the loop, the potential is back where it started.

25.20 Potential rises and drops in a circuit.



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In this section we have considered only situations in which the resistances are ohmic. If the circuit includes a nonlinear device such as a diode (see Fig. 25.10b), Eq. (25.16) is still valid but cannot be solved algebraically because R is not a constant. In such a situation, I can be found by using numerical techniques.

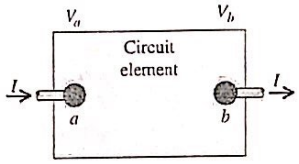
Finally, we remark that Eq. (25.15) is not always an adequate representation of the behavior of a source. The emf may not be constant, and what we have described as an internal resistance may actually be a more complex voltage–current relationship that doesn't obey Ohm's law. Nevertheless, the concept of internal resistance frequently provides an adequate description of batteries, generators, and other energy converters. The principal difference between a fresh flashlight battery and an old one is not in the emf, which decreases only slightly with use, but in the internal resistance, which may increase from less than an ohm when the battery is fresh to as much as 1000 Ω or more after long use. Similarly, a car battery can deliver less current to the starter motor on a cold morning than when the battery is warm, not because the emf is appreciably less but because the internal resistance increases with decreasing temperature.

TEST YOUR UNDERSTANDING OF SECTION 25.4 Rank the following circuits in order from highest to lowest current: (i) A 1.4-Ω resistor connected to a 1.5-V battery that has an internal resistance of 0.10 Ω; (ii) a 1.8-Ω resistor connected to a 4.0-V battery that has a terminal voltage of 3.6 V but an unknown internal resistance; (iii) an unknown resistor connected to a 12.0-V battery that has an internal resistance of 0.20 Ω and a terminal voltage of 11.0 V. **I**

25.5 ENERGY AND POWER IN ELECTRIC CIRCUITS

Let's now look at some energy and power relationships in electric circuits. The box in **Fig. 25.21** represents a circuit element with potential difference $V_a - V_b = V_{ab}$ between its terminals and current I passing through it in the direction from a toward b. This element might be a resistor, a battery, or something else; the details don't matter. As charge passes through the circuit element, the electric field does work on the charge. In a source of emf, additional work is done by the force $\vec{F}_n$ that we mentioned in Section 25.4.

25.21 The power input to the circuit element between a and b is $P = (V_a - V_b)I = V_{ab}I$.



MP

PhET: Battery-Resistor Circuit
PhET: Circuit Construction Kit (AC+DC)
PhET: Circuit Construction Kit (DC Only)
PhET: Ohm's Law

As an amount of charge q passes through the circuit element, there is a change in potential energy equal to qV_{ab}. For example, if $q > 0$ and $V_{ab} = V_a - V_b$ is positive, potential energy decreases as the charge "falls" from potential V_a to lower potential V_b. The moving charges don't gain *kinetic* energy, because the current (the rate of charge flow) out of the circuit element must be the same as the current into the element. Instead, the quantity qV_{ab} represents energy transferred into the circuit element. This situation occurs in the coils of a toaster or electric oven, in which electrical energy is converted to thermal energy.

If the potential at a is lower than at b, then V_{ab} is negative and there is a net transfer of energy *out* of the circuit element. The element then acts as a source, delivering electrical energy into the circuit to which it is attached. This is the usual situation for a battery, which converts chemical energy into electrical energy and delivers it to the external circuit. Thus qV_{ab} can denote a quantity of energy that is either delivered to a circuit element or extracted from that element.

In electric circuits we are most often interested in the *rate* at which energy is either delivered to or extracted from a circuit element. If the current through the element is I, then in a time interval dt an amount of charge $dQ = I\,dt$ passes through the element. The potential energy change for this amount of charge is $V_{ab}\,dQ = V_{ab}I\,dt$. Dividing this expression by dt, we obtain the *rate* at which energy is transferred either into or out of the circuit element. The time rate of energy transfer is *power*, denoted by P, so we write

$$P = V_{ab}I \qquad (25.17)$$

Power delivered to or extracted from a circuit element = Voltage across circuit element × Current in circuit element

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The unit of V_{ab} is one volt, or one joule per coulomb, and the unit of I is one ampere, or one coulomb per second. Hence the unit of $P = V_{ab}I$ is one watt:

$$(1\ \text{J/C})(1\ \text{C/s}) = 1\ \text{J/s} = 1\ \text{W}$$

Let's consider a few special cases.

Power Input to a Pure Resistance

If the circuit element in Fig. 25.21 is a resistor, the potential difference is $V_{ab} = IR$. From Eq. (25.17) the power delivered to the resistor by the circuit is

Power delivered to a resistor: $P = V_{ab}I = I^2R = \dfrac{V_{ab}^2}{R}$ (25.18)

Voltage across resistor; Current in resistor; Resistance of resistor

DEMO

In this case the potential at a (where the current enters the resistor) is always higher than that at b (where the current exits). Current enters the higher-potential terminal of the device, and Eq. (25.18) represents the rate of transfer of electric potential energy *into* the circuit element.

What becomes of this energy? The moving charges collide with atoms in the resistor and transfer some of their energy to these atoms, increasing the internal energy of the material. Either the temperature of the resistor increases or there is a flow of heat out of it, or both. In any of these cases we say that energy is *dissipated* in the resistor at a rate I^2R. Every resistor has a *power rating*, the maximum power the device can dissipate without becoming overheated and damaged. Some devices, such as electric heaters, are designed to get hot and transfer heat to their surroundings. But if the power rating is exceeded, even such a device may melt or even explode.

Power Output of a Source

The upper rectangle in **Fig. 25.22a** represents a source with emf $\mathcal{E}$ and internal resistance r, connected by ideal (resistanceless) conductors to an external circuit represented by the lower box. This could describe a car battery connected to one of the car's headlights (Fig. 25.22b). Point a is at higher potential than point b, so $V_a > V_b$ and V_{ab} is positive. Note that the current I is *leaving* the source at the higher-potential terminal (rather than entering there). Energy is being delivered to the external circuit, at a rate given by Eq. (25.17):

$$P = V_{ab}I$$

For a source that can be described by an emf $\mathcal{E}$ and an internal resistance r, we may use Eq. (25.15):

$$V_{ab} = \mathcal{E} - Ir$$

Multiplying this equation by I, we find

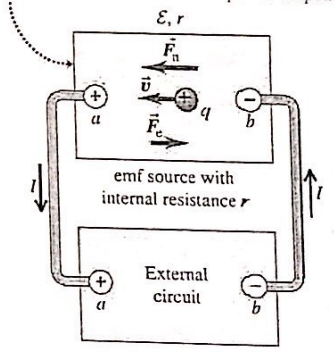
$$P = V_{ab}I = \mathcal{E}I - I^2r \qquad (25.19)$$

What do the terms $\mathcal{E}I$ and I^2r mean? In Section 25.4 we defined the emf $\mathcal{E}$ as the work per unit charge performed on the charges by the nonelectrostatic force as the charges are pushed "uphill" from b to a in the source. In a time dt, a charge $dQ = I\,dt$ flows through the source; the work done on it by this nonelectrostatic force is $\mathcal{E}\,dQ = \mathcal{E}I\,dt$. Thus $\mathcal{E}I$ is the *rate* at which work is done on the circulating charges by whatever agency causes the nonelectrostatic force in the source. This term represents the rate of conversion of nonelectrical energy to electrical energy within the source. The term I^2r is the rate at which electrical energy is

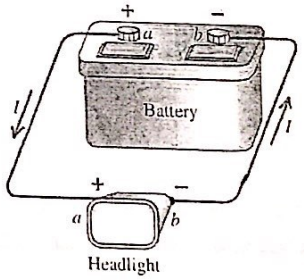
25.22 Energy conversion in a simple circuit.

(a) Diagrammatic circuit

- The emf source converts nonelectrical to electrical energy at a rate $\mathcal{E}I$.
- Its internal resistance *dissipates* energy at a rate I^2r.
- The difference $\mathcal{E}I - I^2r$ is its power output.



(b) A real circuit of the type shown in (a)



Headlight

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dissipated in the internal resistance of the source. The difference $\mathcal{E}I - I^2r$ is the *net* electrical power output of the source—that is, the rate at which the source delivers electrical energy to the remainder of the circuit.

Power Input to a Source

Suppose that the lower rectangle in Fig. 25.22a is itself a source, with an emf *larger* than that of the upper source and opposite to that of the upper source. **Figure 25.23** shows a practical example, an automobile battery (the upper circuit element) being charged by the car's alternator (the lower element). The current I in the circuit is then *opposite* to that shown in Fig. 25.22; the lower source is pushing current backward through the upper source. Because of this reversal of current, instead of Eq. (25.15), we have for the upper source

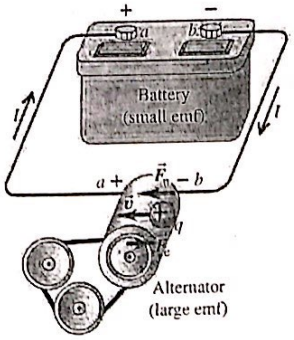
$$V_{ab} = \mathcal{E} + Ir$$

and instead of Eq. (25.19), we have

$$P = V_{ab}I = \mathcal{E}I + I^2r \tag{25.20}$$

Work is being done *on*, rather than *by*, the agent that causes the nonelectrostatic force in the upper source. There is a conversion of electrical energy into nonelectrical energy in the upper source at a rate $\mathcal{E}I$. The term I^2r in Eq. (25.20) is again the rate of dissipation of energy in the internal resistance of the upper source, and the sum $\mathcal{E}I + I^2r$ is the total electrical power *input* to the upper source. This is what happens when a rechargeable battery (a storage battery) is connected to a charger. The charger supplies electrical energy to the battery; part of it is converted to chemical energy, to be reconverted later, and the remainder is dissipated (wasted) in the battery's internal resistance, warming the battery and causing a heat flow out of it. If you have a power tool or laptop computer with a rechargeable battery, you may have noticed that it gets warm while it is charging.

25.23 When two sources are connected in a simple loop circuit, the source with the larger emf delivers energy to the other source.



PROBLEM-SOLVING STRATEGY 25.1 POWER AND ENERGY IN CIRCUITS

IDENTIFY *the relevant concepts:* The ideas of electrical power input and output can be applied to any electric circuit. Many problems will ask you to explicitly consider power or energy.

SET UP *the problem* using the following steps:

1. Make a drawing of the circuit.
2. Identify the circuit elements, including sources of emf and resistors. We will introduce other circuit elements later, including capacitors (Chapter 26) and inductors (Chapter 30).
3. Identify the target variables. Typically they will be the power input or output for each circuit element, or the total amount of energy put into or taken out of a circuit element in a given time.

EXECUTE *the solution* as follows:

1. A source of emf $\mathcal{E}$ delivers power $\mathcal{E}I$ into a circuit when current I flows through the source in the direction from − to +. (For example, energy is converted from chemical energy in a battery, or from mechanical energy in a generator.) In this case there is a *positive* power output to the circuit or, equivalently, a *negative* power input to the source.
2. A source of emf takes power $\mathcal{E}I$ from a circuit when current passes through the source from + to −. (This occurs in charging a storage battery, when electrical energy is converted to chemical energy.) In this case there is a *negative* power output to the circuit or, equivalently, a *positive* power input to the source.
3. There is always a *positive* power input to a resistor through which current flows, irrespective of the direction of current flow. This process removes energy from the circuit, converting it to heat at the rate $VI = I^2R = V^2/R$, where V is the potential difference across the resistor.
4. Just as in item 3, there always is a positive power input to the internal resistance r of a source through which current flows, irrespective of the direction of current flow. This process likewise removes energy from the circuit, converting it into heat at the rate I^2r.
5. If the power into or out of a circuit element is constant, the energy delivered to or extracted from that element is the product of power and elapsed time. (In Chapter 26 we will encounter situations in which the power is not constant. In such cases, calculating the total energy requires an integral over the relevant time interval.)

EVALUATE *your answer:* Check your results; in particular, check that energy is conserved. This conservation can be expressed in either of two forms: "net power input = net power output" or "the algebraic sum of the power inputs to the circuit elements is zero."

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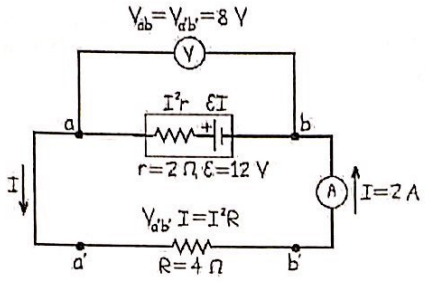
EXAMPLE 25.8 POWER INPUT AND OUTPUT IN A COMPLETE CIRCUIT

For the circuit that we analyzed in Example 25.5, find the rates of energy conversion (chemical to electrical) and energy dissipation in the battery, the rate of energy dissipation in the 4-Ω resistor, and the battery's net power output.

SOLUTION

IDENTIFY and SET UP: **Figure 25.24** shows the circuit, gives values of quantities known from Example 25.5, and indicates how we find the target variables. We use Eq. (25.19) to find the battery's net power output, the rate of chemical-to-electrical energy conversion, and the rate of energy dissipation in the battery's internal resistance. We use Eq. (25.18) to find the power delivered to (and dissipated in) the 4-Ω resistor.

25.24 Our sketch for this problem.



EXECUTE: From the first term in Eq. (25.19), the rate of energy conversion in the battery is

$$\mathcal{E}I = (12\text{ V})(2\text{ A}) = 24\text{ W}$$

From the second term in Eq. (25.19), the rate of dissipation of energy in the battery is

$$I^2r = (2\text{ A})^2(2\ \Omega) = 8\text{ W}$$

The *net* electrical power output of the battery is the difference between these: $\mathcal{E}I - I^2r = 16$ W. From Eq. (25.18), the electrical power input to, and the equal rate of dissipation of electrical energy in, the 4-Ω resistor are

$$V_{a'b'}I = (8\text{ V})(2\text{ A}) = 16\text{ W} \quad \text{and}$$

$$I^2R = (2\text{ A})^2(4\ \Omega) = 16\text{ W}$$

EVALUATE: The rate $V_{a'b'}I$ at which energy is supplied to the 4-Ω resistor equals the rate I^2R at which energy is dissipated there. This is also equal to the battery's net power output: $P = V_{ab}I = (8\text{ V})(2\text{ A}) = 16$ W. In summary, the rate at which the source of emf supplies energy is $\mathcal{E}I = 24$ W, of which $I^2r = 8$ W is dissipated in the battery's internal resistor and $I^2R = 16$ W is dissipated in the external resistor.

EXAMPLE 25.9 INCREASING THE RESISTANCE

Suppose we replace the external 4-Ω resistor in Fig. 25.24 with an 8-Ω resistor. How does this affect the electrical power dissipated in this resistor?

SOLUTION

IDENTIFY and SET UP: Our target variable is the power dissipated in the resistor to which the battery is connected. The situation is the same as in Example 25.8, but with a higher external resistance R.

EXECUTE: According to Eq. (25.18), the power dissipated in the resistor is $P = I^2R$. You might conclude that making the resistance R twice as great as in Example 25.8 should make the power twice as great, or $2(16\text{ W}) = 32$ W. If instead you used the formula $P = V_{ab}^2/R$, you might conclude that the power should be one-half as great as in the preceding example, or $(16\text{ W})/2 = 8$ W. Which answer is correct?

In fact, *both* of these answers are *incorrect*. The first is wrong because changing the resistance R also changes the current in the circuit (remember, a source of emf does *not* generate the same current in all situations). The second answer is wrong because the potential difference V_{ab} across the resistor changes when the current changes. To get the correct answer, we first find the current just as we did in Example 25.5:

$$I = \frac{\mathcal{E}}{R + r} = \frac{12\text{ V}}{8\ \Omega + 2\ \Omega} = 1.2\text{ A}$$

The greater resistance causes the current to decrease. The potential difference across the resistor is

$$V_{ab} = IR = (1.2\text{ A})(8\ \Omega) = 9.6\text{ V}$$

which is greater than that with the 4-Ω resistor. We can then find the power dissipated in the resistor in either of two ways:

$$P = I^2R = (1.2\text{ A})^2(8\ \Omega) = 12\text{ W} \quad \text{or}$$

$$P = \frac{V_{ab}^2}{R} = \frac{(9.6\text{ V})^2}{8\ \Omega} = 12\text{ W}$$

EVALUATE: Increasing the resistance R causes a *reduction* in the power input to the resistor. In the expression $P = I^2R$ the decrease in current is more important than the increase in resistance; in the expression $P = V_{ab}^2/R$ the increase in resistance is more important than the increase in V_{ab}. This same principle applies to ordinary light bulbs; a 50-W light bulb has a greater resistance than does a 100-W light bulb.

Can you show that replacing the 4-Ω resistor with an 8-Ω resistor decreases both the rate of energy conversion (chemical to electrical) in the battery and the rate of energy dissipation in the battery?

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EXAMPLE 25.10 POWER IN A SHORT CIRCUIT

SOLUTION

For the short-circuit situation of Example 25.7, find the rates of energy conversion and energy dissipation in the battery and the net power output of the battery.

SOLUTION

IDENTIFY and SET UP: Our target variables are again the power inputs and outputs associated with the battery. **Figure 25.25** shows the circuit. This is the same situation as in Example 25.8, but now the external resistance R is zero.

EXECUTE: We found in Example 25.7 that the current in this situation is $I = 6$ A. From Eq. (25.19), the rate of energy conversion (chemical to electrical) in the battery is then

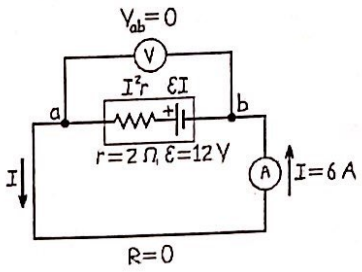
$$\mathcal{E}I = (12\text{ V})(6\text{ A}) = 72\text{ W}$$

and the rate of dissipation of energy in the battery is

$$I^2r = (6\text{ A})^2(2\ \Omega) = 72\text{ W}$$

The net power output of the source is $\mathcal{E}I - I^2r = 0$. We get this same result from the expression $P = V_{ab}I$, because the terminal voltage V_{ab} of the source is zero.

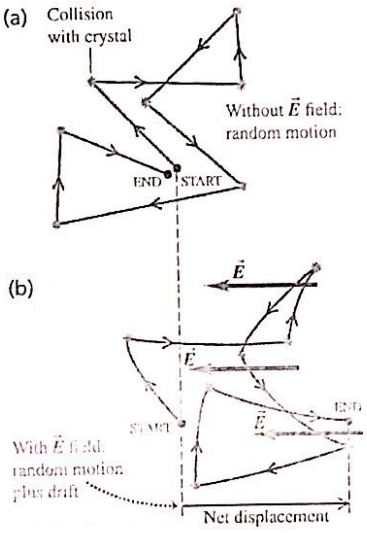
25.25 Our sketch for this problem.



EVALUATE: With ideal wires and an ideal ammeter, so that $R = 0$, *all* of the converted energy from the source is dissipated within the source. This is why a short-circuited battery is quickly ruined and may explode.

TEST YOUR UNDERSTANDING OF SECTION 25.5 Rank the following circuits in order from highest to lowest values of the net power output of the battery. (i) A 1.4-Ω resistor connected to a 1.5-V battery that has an internal resistance of 0.10 Ω; (ii) a 1.8-Ω resistor connected to a 4.0-V battery that has a terminal voltage of 3.6 V but an unknown internal resistance; (iii) an unknown resistor connected to a 12.0-V battery that has an internal resistance of 0.20 Ω and a terminal voltage of 11.0 V. ❙

25.26 Random motions of an electron in a metallic crystal (a) with zero electric field and (b) with an electric field that causes drift. The curvatures of the paths are greatly exaggerated.



25.6 THEORY OF METALLIC CONDUCTION

We can gain additional insight into electrical conduction by looking at the microscopic origin of conductivity. We'll consider a very simple model that treats the electrons as classical particles and ignores their quantum-mechanical behavior in solids. Using this model, we'll derive an expression for the resistivity of a metal. Even though this model is not entirely correct, it will still help you to develop an intuitive idea of the microscopic basis of conduction.

In the simplest microscopic model of conduction in a metal, each atom in the metallic crystal gives up one or more of its outer electrons. These electrons are then free to move through the crystal, colliding at intervals with the stationary positive ions. The motion of the electrons is analogous to the motion of molecules of a gas moving through a porous bed of sand.

If there is no electric field, the electrons move in straight lines between collisions, the directions of their velocities are random, and on average they never get anywhere (**Fig. 25.26a**). But if an electric field is present, the paths curve slightly because of the acceleration caused by electric-field forces. Figure 25.26b shows a few paths of an electron in an electric field directed from right to left. As we mentioned in Section 25.1, the average speed of random motion is of the order of 10^6 m/s, while the average drift speed is *much* slower, of the order of 10^{-4} m/s.

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The average time between collisions is called the **mean free time,** denoted by τ. **Figure 25.27** shows a mechanical analog of this electron motion.

We would like to derive from this model an expression for the resistivity ρ of a material, defined by Eq. (25.5):

$$\rho = \frac{E}{J} \tag{25.21}$$

where E and J are the magnitudes of electric field and current density, respectively. The current density $\vec{J}$ is in turn given by Eq. (25.4):

$$\vec{J} = nq\vec{v}_d \tag{25.22}$$

where n is the number of free electrons per unit volume (the electron concentration), $q = -e$ is the charge of each, and $\vec{v}_d$ is their average drift velocity.

We need to relate the drift velocity $\vec{v}_d$ to the electric field $\vec{E}$. The value of $\vec{v}_d$ is determined by a steady-state condition in which, on average, the velocity *gains* of the charges due to the force of the $\vec{E}$ field are just balanced by the velocity *losses* due to collisions. To clarify this process, let's imagine turning on the two effects one at a time. Suppose that before time $t = 0$ there is no field. The electron motion is then completely random. A typical electron has velocity $\vec{v}_0$ at time $t = 0$, and the value of $\vec{v}_0$ averaged over many electrons (that is, the initial velocity of an average electron) is zero: $(\vec{v}_0)_{av} = 0$. Then at time $t = 0$ we turn on a constant electric field $\vec{E}$. The field exerts a force $\vec{F} = q\vec{E}$ on each charge, and this causes an acceleration $\vec{a}$ in the direction of the force, given by

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

where m is the electron mass. *Every* electron has this acceleration.

After a time τ, the average time between collisions, we "turn on" the collisions. At time $t = \tau$ an electron that has velocity $\vec{v}_0$ at time $t = 0$ has a velocity

$$\vec{v} = \vec{v}_0 + \vec{a}\tau$$

The velocity $\vec{v}_{av}$ of an *average* electron at this time is the sum of the averages of the two terms on the right. As we have pointed out, the initial velocity $\vec{v}_0$ is zero for an average electron, so

$$\vec{v}_{av} = \vec{a}\tau = \frac{q\tau}{m}\vec{E} \tag{25.23}$$

After time $t = \tau$, the tendency of the collisions to decrease the velocity of an average electron (by means of randomizing collisions) just balances the tendency of the $\vec{E}$ field to increase this velocity. Thus the velocity of an average electron, given by Eq. (25.23), is maintained over time and is equal to the drift velocity $\vec{v}_d$:

$$\vec{v}_d = \frac{q\tau}{m}\vec{E}$$

Now we substitute this equation for the drift velocity $\vec{v}_d$ into Eq. (25.22):

$$\vec{J} = nq\vec{v}_d = \frac{nq^2\tau}{m}\vec{E}$$

Comparing this with Eq. (25.21), which we can rewrite as $\vec{J} = \vec{E}/\rho$, and substituting $q = -e$ for an electron, we see that

$$\rho = \frac{m}{ne^2\tau} \tag{25.24}$$

Resistivity of a metal

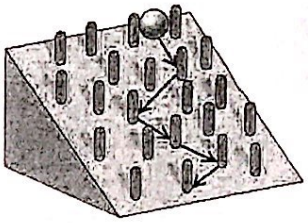
Number of free electrons per unit volume

Electron mass

Average time between collisions

Magnitude of electron charge

25.27 The motion of a ball rolling down an inclined plane and bouncing off pegs in its path is analogous to the motion of an electron in a metallic conductor with an electric field present.



MP

PhET: Conductivity

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If n and τ are independent of $\vec{E}$, then the resistivity is independent of $\vec{E}$ and the conducting material obeys Ohm's law.

Turning the interactions on one at a time may seem artificial. But the derivation would come out the same if each electron had its own clock and the $t = 0$ times were different for different electrons. If τ is the average time between collisions, then $\vec{v}_d$ is still the average electron drift velocity, even though the motions of the various electrons aren't actually correlated in the way we postulated.

What about the temperature dependence of resistivity? In a perfect crystal with no atoms out of place, a correct quantum-mechanical analysis would let the free electrons move through the crystal with no collisions at all. But the atoms vibrate about their equilibrium positions. As the temperature increases, the amplitudes of these vibrations increase, collisions become more frequent, and the mean free time τ decreases. So this theory predicts that the resistivity of a metal increases with temperature. In a superconductor, roughly speaking, there are no inelastic collisions, τ is infinite, and the resistivity ρ is zero.

In a pure semiconductor such as silicon or germanium, the number of charge carriers per unit volume, n, is not constant but increases very rapidly with increasing temperature. This increase in n far outweighs the decrease in the mean free time, and in a semiconductor the resistivity always decreases rapidly with increasing temperature. At low temperatures, n is very small, and the resistivity becomes so large that the material can be considered an insulator.

Electrons gain energy between collisions through the work done on them by the electric field. During collisions they transfer some of this energy to the atoms of the material of the conductor. This leads to an increase in the material's internal energy and temperature; that's why wires carrying current get warm. If the electric field in the material is large enough, an electron can gain enough energy between collisions to knock off electrons that are normally bound to atoms in the material. These can then knock off more electrons, and so on, leading to an avalanche of current. This is the basis of dielectric breakdown in insulators (see Section 24.4).

EXAMPLE 25.11 MEAN FREE TIME IN COPPER

Calculate the mean free time between collisions in copper at room temperature.

SOLUTION

IDENTIFY and SET UP: We can obtain an expression for mean free time τ in terms of n, ρ, e, and m by rearranging Eq. (25.24). From Example 25.1 and Table 25.1, for copper $n = 8.5 \times 10^{28}\ \text{m}^{-3}$ and $\rho = 1.72 \times 10^{-8}\ \Omega \cdot \text{m}$. In addition, $e = 1.60 \times 10^{-19}\ \text{C}$ and $m = 9.11 \times 10^{-31}\ \text{kg}$ for electrons.

EXECUTE: From Eq. (25.24), we get

$$\tau = \frac{m}{ne^2\rho}$$

$$= \frac{9.11 \times 10^{-31}\ \text{kg}}{(8.5 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})^2(1.72 \times 10^{-8}\ \Omega \cdot \text{m})}$$

$$= 2.4 \times 10^{-14}\ \text{s}$$

EVALUATE: The mean free time is the average time between collisions for a given electron. Taking the reciprocal, we find that each electron averages $1/\tau = 4.2 \times 10^{13}$ collisions per second!

TEST YOUR UNDERSTANDING OF SECTION 25.6 Which of the following factors will, if increased, make it more difficult to produce a certain amount of current in a conductor? (There may be more than one correct answer.) (i) The mass of the moving charged particles in the conductor; (ii) the number of moving charged particles per cubic meter; (iii) the amount of charge on each moving particle; (iv) the average time between collisions for a typical moving charged particle. ❙

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25.1 IC

25.2
$I = 0.09A$
$v_d = 1.78 \times 10^{-6}\ m/s$

25.3
$dQ = S(1) = SC$
$J = 1.5 \times 10^{6}\ A/m^2$
$v_d = 1.1 \times 10^{-4}\ m/s$

24.4
$I = 2.6A$ $v_d = 2.3 \times 10^{-4}\ m/s$

25.5
a) $6.6 \times 10^{3}\ S$ b) $2.7 \times 10^{4}\ S$

25.6
$L_G = 32.2$
$L_C = 45.7$
$L_A = 28.6$
$m_G = 0.49\ kg$

25.7
$Q = dJ/I = 330C$
$I = \frac{330}{8} = 41A$

25.8
$Q = qn = 0.0106C$ $I = 0.0106\ A$

25.10
$E_C = 0.023\ V/m$
$E_S = 0.02\ V/m$

25.11
$\rho_0 = 1.05 \times 10^{-5}\ \Omega m$
$\alpha = 1.7 \times 10^{-3}\ (°C)^{-1}$

25.12
$J = 6.8 \times 10^{5}\ A/m^2$
$dt = 8 \times 10^{4}\ s$ $E = 0.012\ V/m$

SOLUTION

A toaster using a Nichrome heating element operates on 120 V.

3. The power P dissipated in the heating element depends on I

to calculate the

a relationship

initial and final

itial and final

an 20°C? Does

8. Is the final resistance greater than or less than the initial resis-

he initial power?

8?

that it carries.

25.13
a) $2.125 \times 10^{-4}\ V$
b) $1.75 \times 10^{-4}\ V$

25.14
1.47×10^{-8} Silver

25.15
a) $1.216\ V/m$
b) $0.0145\ \Omega$
c) $V = 0.181V$

25.16
$R = \frac{\rho L}{A}$
$R_{new} = \frac{\rho 3L}{\frac{A}{3}}$
$= 9\frac{\rho L}{A}$
$= 9R_{old}$

$V = V_{new}$
$LA = L_{new} A_{new}$
$A_{new} = \frac{L}{3L} A$
$= \frac{A}{3}$

25.17 $R = 0.125\ \Omega$

25.18
$R_C = R_A$
$\rho_C \frac{L}{\pi r_C^2} = \frac{\rho_A L}{\pi r_A^2}$
$r_C^2 = \sqrt{\frac{\rho_C r_A^2}{\rho_A}}$
$= 8.5 \times 10^{-4}\ m \xrightarrow{\times 2} d = 1.7mm$

25.19
a) increase $A \rightarrow$ decrease R
$\frac{R}{120} = 4.67 \times 10^{-8}\ \Omega$
b) increase $L \rightarrow$ increase R
$R(120) = 6.72 \times 10^{-4}\ \Omega$

24.20 $1.4 \times 10^{-7}\ \Omega m$

24.21
$I = 11A$ $V = 3.2V$ $R = 0.29\ \Omega$

25.22
a) $A = \pi(r_b^2 - r_a^2)$
$R = 2 \times 10^{-5}\ \Omega$
b) $dR = \frac{\rho dr}{2\pi r L}$
$R = \frac{\rho}{2\pi L} \ln\left(\frac{r_b}{r_a}\right)$
$= 6.35 \times 10^{-10}\ \Omega$

25.23
a) $99.5\ \Omega$
b) $0.016\ \Omega$

25.24
$T = 17.8°C$

25.25
$R = 0.219\ \Omega$
$V = 27.4V$
$P = 3422\ J/s \rightarrow 1.23 \times 10^{7}\ J/h$

Physics". MP

•, ••, •••: Difficulty levels. CP: Cumulative problems incorporating material from earlier chapters. CALC: Problems requiring calculus.

s problems.

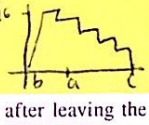
25.26
a) $\Delta r = 0.7$
$R = 5.3$

25.27
a) No current
b) $\mathcal{E} = V = 5V$
c) $5V$

25.28
a) $I = \mathcal{E}/r = 5A$
b) No current, $V = 0$
c) zero

25.29
$\mathcal{E} = 3.38V$
$r = 0.07\ \Omega$
$R = 1.8\ \Omega$

25.30
$I = \frac{16-8}{1.6+5+1.4+9} = 0.47A$
b) $V_b + 16 - I(1.6) = V_a$
$V_a - V_b = 15.2V$
c) $V_c + 8 + I(1.4+5) = V_a$
$V_{ac} = 11\ V$
$V_{cb} = 4.2V$



nf; for instance,
Would it also be
w much current

nous amount of
ge of 1 C on an
sonable. Explain

rough a resistor.
same diameter.
ore entering the

smaller-diameter wire? Does their drift speed increase, decrease, or stay the same? If the drift speed changes, what is the force that

resistor compare to the speed after leaving the resistor? Explain your reasoning. (b) How does the potential energy for an electron

l energy after

are given in
onnected to a
wer consumed
s temperature
arbon cylinder
erature of the
umes increase

25.31
a) $I = \frac{16+8}{10} = 1.4\ A$
b) $V_a + 16 - I(1.6) = V_b$
$V_{ab} = -13.7\ V$
$V_{ba} = 13.7V$
c) $V_{ac} = -1V$ $V_{ca} = 1V$

25.32
a) $I = \frac{V}{R} = \frac{1.2}{5} = 0.21A$
b) $I = \frac{\mathcal{E}}{R+r}$ $R = \frac{\mathcal{E}}{I} - r$
$= 26.1\ \Omega$
c)

25.33
$I = 0.47A$
$V_{ab} = 15.2V$

25.34
$R = \frac{V^2}{P} = 36\ \Omega$
a) $4.41\ W$ b) $13.42\ V$

25.35
a) $144\ \Omega$ b) $243\ \Omega$
c) $0.8A$, $0.5A$

25.36
$\frac{(120)^2}{75} = 192\ \Omega$
$\frac{(220)^2}{192} = 252\ W$

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25.37
$R = 484\ \Omega$
$P = 29.8\ W$
$I = 0.25\ A$

25.38
$P = 1.17\ J/s$
$1.17(1800) = 2106\ J$

25.39
a) $P_S = 1.1\ W$ b) 7.2W
$P_3 = 1.79\ W$ c) 4.1 W
d) $1.4 + 1.79 + 1 = 7.2$

25.42
$I = \frac{\mathcal{E}}{TV} = 0.45A$

25.43
$60(12) = 720\ J/s$
$720(60)(60) = 2.6 \times 10^6\ J$
$P = \frac{E}{T}$ $T = 5778s = 1.6h$

25.44
$r = 12.8\ \Omega$
$P = 2.2W$

25.45
$\frac{P}{P_{total}} = \frac{I^2 r}{I^2(R+r)} = \frac{3.5}{25+3.5} = 12.3\%$

25.46
$I = 0.18A$ $E = 7700\ J$
$P = 0.55W$ $r = 6.1\ \Omega$

25.47
$I = 2A$
a) $P = \mathcal{E}I = 24W$
b) $Ir^2 = 4W$
c) $IR^2 = 20W$

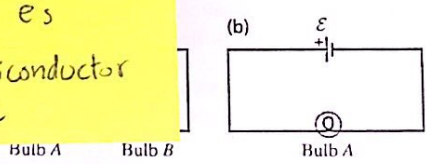
Figure Q25.12

25.48
a) $P = V^2/R$ $R = \frac{V^2}{P} = 26.7\ \Omega$
b) $P = VI \Rightarrow I = P/V$
$= 4.5\ A$
c) $R = 26.7\ \Omega$
$P = \frac{V^2}{R} = \frac{(110)^2}{26.7} = 453W$

25.49 $\rho = \frac{m}{ne^2\tau}$
$\rho = 2300\ \Omega m$
a) $\tau = \frac{m}{ne^2\rho} = 1.55 \times 10^{-12}\ s$
b) Co free e's
Si Semiconductor
NTC

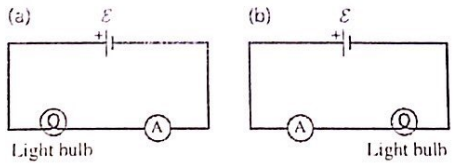
ctric light bulb nearly always burn
light, almost never while the light is

because it has resistance. The bright-
ness of a light bulb increases with the electrical power dissipated
in the bulb. (a) In the circuit shown in **Fig. Q25.14a**, the two bulbs
red to bulb *A*, does bulb *B* glow more
ess brightly? Explain your reasoning.
he circuit and the circuit is completed
mpared to the brightness of bulb *A* in
v glow more brightly, just as brightly,
reasoning.



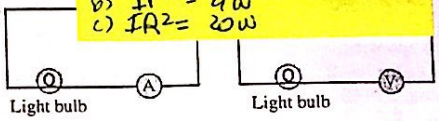
Q25.15 (See Discussion Question Q25.14.) An ideal ammeter A is placed in a circuit with a battery and a light bulb as shown in **Fig. Q25.15a**, and the ammeter reading is noted. The circuit is then reconnected as in Fig. Q25.15b, so that the positions of the ammeter and light bulb are reversed. (a) How does the ammeter reading in the situation shown in Fig. Q25.15a compare to the reading in the situation shown in Fig. Q25.15b? Explain your reasoning. (b) In which situation does the light bulb glow more brightly? Explain your reasoning.

Figure Q25.15



Q25.16 (See Discussion Question Q25.14.) Will a light bulb glow more brightly when it is connected to a battery as shown in **Fig. Q25.16a**, in which an ideal ammeter A is placed in the circuit, or when it is connected as shown in Fig. 25.16b, in which an ideal voltmeter V is placed in the circuit? Explain your reasoning.

Q25.16
(a)



Q25.17 The energy that can be extracted from a storage battery is always less than the energy that goes into it while it is being charged. Why?

Q25.18 Eight flashlight batteries in series have an emf of about 12 V, similar to that of a car battery. Could they be used to start a car with a dead battery? Why or why not?

Q25.19 Small aircraft often have 24-V electrical systems rather than the 12-V systems in automobiles, even though the electrical power requirements are roughly the same in both applications. The explanation given by aircraft designers is that a 24-V system weighs less than a 12-V system because thinner wires can be used. Explain why this is so.

Q25.20 Long-distance, electric-power, transmission lines always operate at very high voltage, sometimes as much as 750 kV. What are the advantages of such high voltages? What are the disadvantages?

Q25.21 Ordinary household electric lines in North America usually operate at 120 V. Why is this a desirable voltage, rather than a value considerably larger or smaller? On the other hand, automobiles usually have 12-V electrical systems. Why is this a desirable voltage?

Q25.22 A fuse is a device designed to break a circuit, usually by melting when the current exceeds a certain value. What characteristics should the material of the fuse have?

Q25.23 High-voltage power supplies are sometimes designed intentionally to have rather large internal resistance as a safety precaution. Why is such a power supply with a large internal resistance safer than a supply with the same voltage but lower internal resistance?

Q25.24 The text states that good thermal conductors are also good electrical conductors. If so, why don't the cords used to connect toasters, irons, and similar heat-producing appliances get hot by conduction of heat from the heating element?

EXERCISES

Section 25.1 Current

25.1 · **Lightning Strikes.** During lightning strikes from a cloud to the ground, currents as high as 25,000 A can occur and last for about 40 μs. How much charge is transferred from the cloud to the earth during such a strike?

25.2 · A silver wire 2.6 mm in diameter transfers a charge of 420 C in 80 min. Silver contains 5.8×10^{28} free electrons per cubic meter. (a) What is the current in the wire? (b) What is the magnitude of the drift velocity of the electrons in the wire?

25.3 · A 5.00-A current runs through a 12-gauge copper wire (diameter 2.05 mm) and through a light bulb. Copper has 8.5×10^{28} free electrons per cubic meter. (a) How many electrons pass through the light bulb each second? (b) What is the current density in the wire? (c) At what speed does a typical electron pass by any given point in the wire? (d) If you were to use wire of twice the diameter, which of the above answers would change? Would they increase or decrease?

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